Innovation Lab

# Portfolio factor allocation schemes

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## Executive summary

Factor allocation decisions are becoming a prominent consideration for factor investors. Which factors to allocate to, and in what magnitude, has a significant impact on investment outcomes and should be a key focus of investors. As multi-factor investing grows in popularity, we hope it will evolve from the mere identification of a set of factors to cover the explicit choice of the relative magnitude of each factor's exposure.

In this paper, we seek to aide this evolution as we present:

- three factor allocation schemes, which can deliver balanced exposure to multiple factors
- meaningful insights into the volatility and correlation of factor risk premia and how this can inform factor allocation decisions
- a transparent, non-optimized, bottom-up portfolio construction mechanism that can deliver bottom up factor allocation

# Contents

# 1 Introduction

The decision over which factors to allocate to, and in what magnitude, has a significant impact on investment outcomes and is an important consideration for investors employing a factor approach to achieve investment objectives. It is analogous to the asset allocation decision within a multi-asset context and is particularly relevant given the increasing popularity of factor investing and the growth in multi-factor investment products.

As with asset allocation, different investors have different objectives. For example, investors employing a defensive equity strategy may allocate to defensive factors such as Quality or Low Volatility [1]. Alternatively, investors with views on the likely magnitude of future factor returns may follow a dynamic allocation strategy. Often, investors are just seeking to improve risk-adjusted performance outcomes and will select a combination of factors for this purpose. However, the allocation or size of the relative exposures to each factor is often disregarded, leading to unnecessary risk concentrations.

In this paper, we compare the outcomes of three formal schemes for determining the allocation to a given set of factors. In the absence of specific views on expected factor returns, a logical objective is to diversify across the range of factors. Three potential allocation schemes are:

- Equal Exposure (EE): equal levels of factor exposure
- Risk Exposure (RE): factor exposure is inversely proportional to the volatility of factor returns, i.e., higher exposure to less volatile factors
- Equal Risk Contribution (ERC): factor exposure is determined such that each factor contributes equally to active risk

Each of these schemes corresponds to a well-known stock level portfolio construction process, where weight is allocated to individual stocks to yield equally-weighted [2], risk-weighted [3] and equal-risk contribution [4] portfolios respectively. Here, we note that such allocation schemes are applicable at the level of factors and that the differences between each scheme may be examined through the assumptions made regarding knowledge of future factor returns, volatilities and correlations.

For example, the EE approach attempts to maximize diversification in terms of factor exposure; ERC attempts to maximize diversification in terms of factor risk contributions. The latter requires specific information regarding the volatility of and correlations between factor risk premia. The former implicitly assumes factor return volatilities are equal and uncorrelated with one another. In the event that this is true, EE allocation will also result in equal factor risk contributions.

If volatility is constant across factor risk premia and the risk premia are themselves uncorrelated, then RE collapses to an EE allocation scheme. If there are meaningful differences in correlation between factor premia, then the use of volatility in a RE approach will be insufficient to equalize risk contributions.

In Section 2 of this paper, we examine the empirical behavior of factor premia, while Section 3 outlines a transparent, non-optimized, bottom-up portfolio construction implementation technique for each multi-factor allocation scheme. Section 4 details the results of each factor allocation scheme. In Section 5, we draw our conclusions.

# 2 The volatility and correlation of factor returns

In this section, we examine the empirical behavior of factor returns to identify persistent characteristics that may facilitate our choice of factor allocation scheme. We calculate daily factor returns for Value, Quality, Low Volatility, Size and Momentum for the period between August 1998 and September 2019 for the FTSE Developed universe (see [7] for factor definitions).

Factor returns are derived from daily cross-sectional market capitalization weighted regressions of total USD stock returns on prior month end factor exposures with industry and country effects; this is illustrated in equation (1). These factor returns represent the performance of a minimum residual long /short portfolio with unit active exposure to the factor of choice and zero active exposure to all other factors. The equation they satisfy is:

$$R_i = \alpha + \sum_C \delta_{i\in C} r_C + \sum_f Z_{fi}\, r_f + \varepsilon_i \tag{1}$$

where for stock $i$:

- $R_i$ is the stock return
- $\delta_{i\in C} = \begin{cases} 1 \; if \; i \in C \\ 0 \; if \; i \notin C \end{cases}$ is a dummy variable representing membership of industry or country $C$
- $Z_{fi}$ is the capitalisation-weighted Z-score for factor $f$
- $\varepsilon_i$ is the residual return

The intercept $\alpha$ is equal to the return of the capitalization weighted benchmark, while $r_C$ is the pure country/industry return and $r_f$ is the pure factor return or factor premium of interest.

## 2.1 Factor volatility

Table 1 shows the annualized volatility of factor returns over the period from August 1998 to September 2019. Value and Quality are the least volatile, while Low Volatility and Momentum are the most volatile.

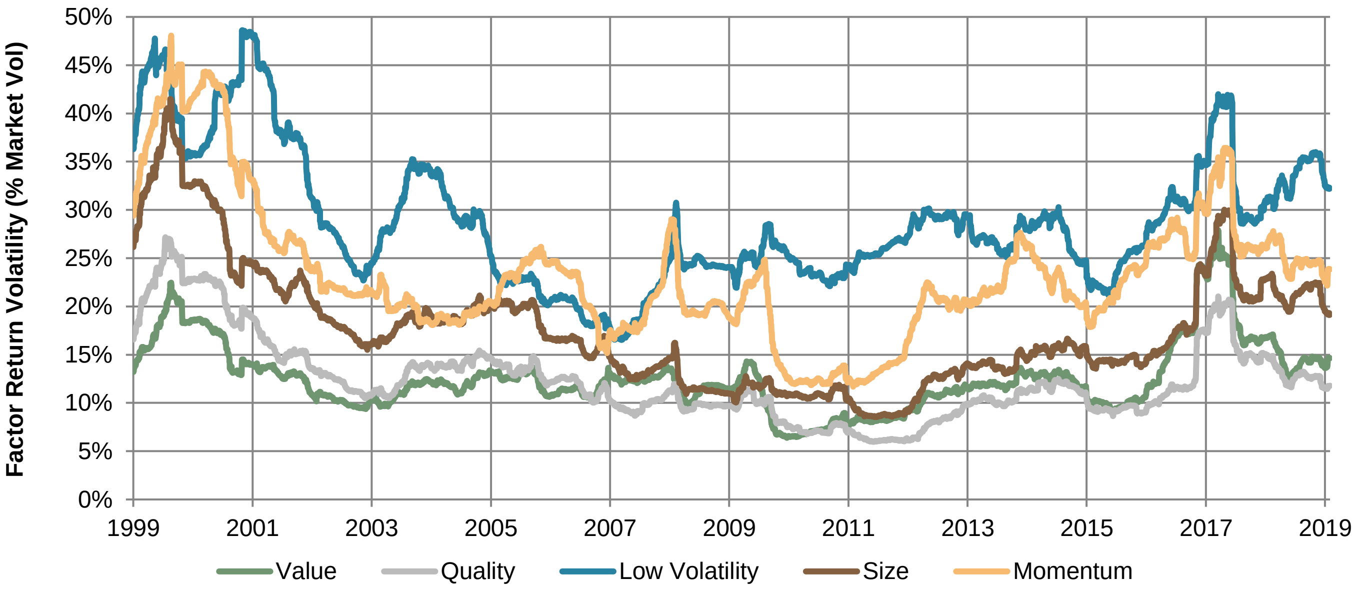
**Table 1: Full Period Volatility (August 1998 - September 2019)**

| Annualized Standard Deviation | | | | | |
|---|---|---|---|---|---|
| Market | Value | Quality | Low Volatility | Size | Momentum |
| 16.02% | 1.95% | 1.99% | 4.65% | 2.79% | 3.79% |

Source: FTSE Russell. Data based on the FTSE Developed Index Universe from August 1998 to September 2019. Past performance is no guarantee of future results. Please see the end for important legal disclosures.

Figure 1 illustrates the rolling one-year factor return volatilities scaled by the market return volatility through time. This demonstrates clear differences in factor returns and considerable variation over the market cycle. The volatility peaks and troughs are unique to each factor, with Low Volatility and Momentum showing the greatest variation.

**Figure 1: Rolling one-year factor return volatility scaled by market volatility**



Source: FTSE Russell. Data based on the FTSE Developed Universe from August 1998 to September 2019. Past performance is no guarantee of future results. Please see the end for important legal disclosures.

Both the RE and ERC allocation schemes employ a measure of volatility to determine the allocation to each factor. A degree of stability or persistence in levels of relative volatility is therefore required if past levels of volatility are to be used as a guide to determining factor allocations. If relative levels of factor return volatility are fast changing and unpredictable, RE and ERC allocation schemes may offer no advantage over simply assuming that factor volatilities are equal, suggesting an EE allocation scheme is applicable.

Figures 2-6 plot the one-year factor return volatility scaled by market volatility against the same figure a year earlier. Empirically, factor-return volatility (when scaled by market volatility) exhibits a degree of persistence, indicating that historic measures of factor volatility are potentially a useful proxy for future levels. This, combined with the consistent differences in volatility, suggest that factor allocation schemes that incorporate volatility, such as RE and ERC, will be superior to an EE scheme in achieving factor diversification.

**Figures 2-6: One-year factor return volatility as a percentage of market volatility**

**Figure 2: Value**

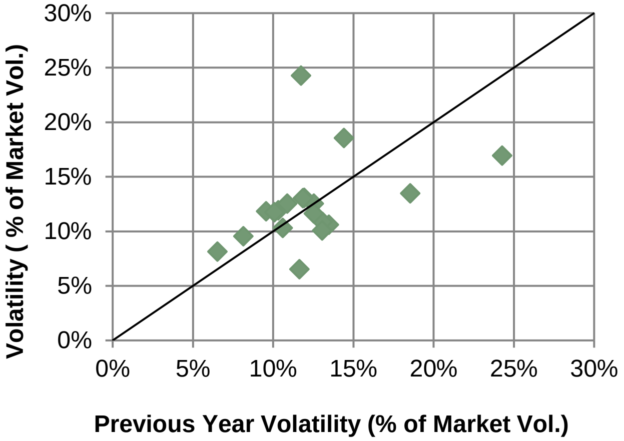


**Figure 3: Quality**

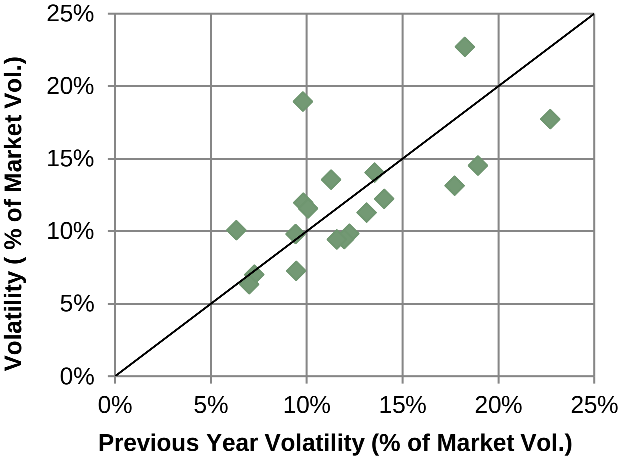


**Figure 4: Low Volatility**

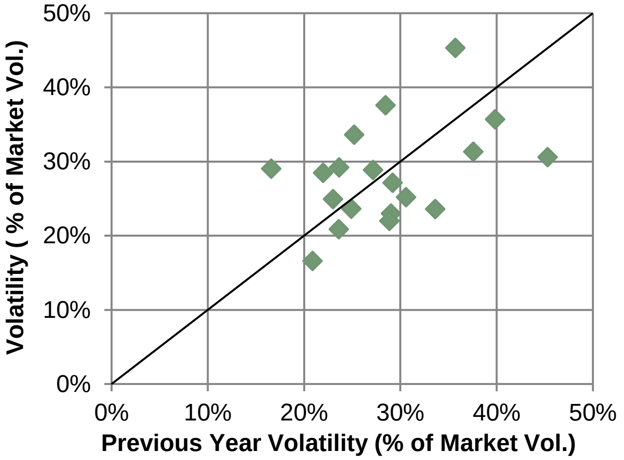


**Figure 5: Size**

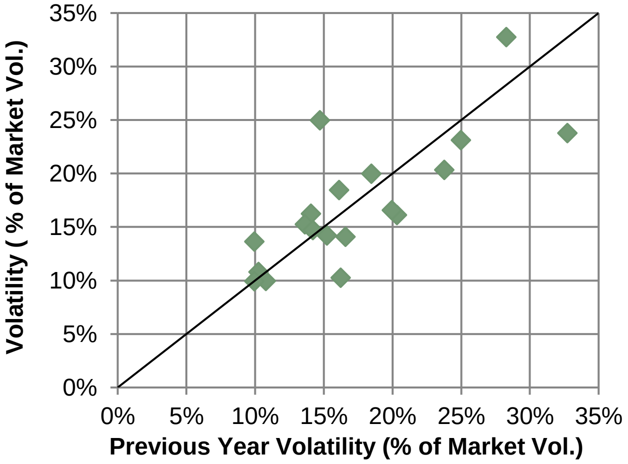
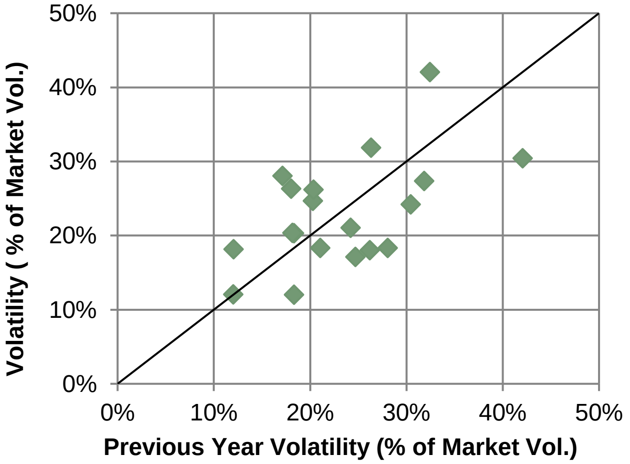


**Figure 6: Momentum**



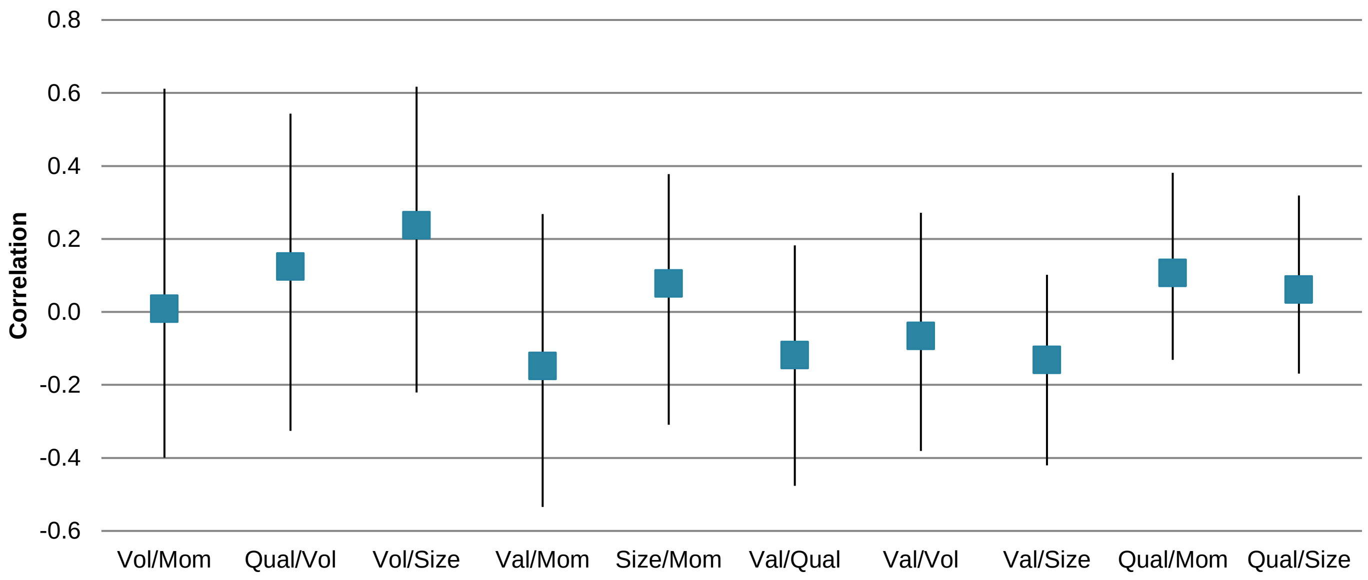
Source: FTSE Russell. Data based on the FTSE Developed Universe from August 1998 to September 2019. Past performance is no guarantee of future results. Please see the end for important legal disclosures.

## 2.2 Factor Correlation

An ERC factor allocation scheme requires, in addition to factor volatility, the specification of the correlations between factor returns. The empirical behavior of the pairwise correlations between factor risk premia is therefore of particular importance.

Figure 7 illustrates the variability of factor return correlations: it displays the maximum, minimum and average one-year correlation of daily factor returns for each factor pair for each year ending in September 1999 through September 2019. Each correlation pair exhibits periods of both positive and negative correlation. The factor pairs are plotted from left to right in increasing order of stability of correlation. The Low Volatility/Momentum pair on the left of Figure 7 is the least stable pair, while Quality/Size is the most stable pair. The levels of variability observed contrast those observed for factor volatilities, which were relatively stable over the period.

**Figure 7: Minimum, maximum and average factor return correlation**



Source: FTSE Russell. Data based on the FTSE Developed Index Universe from August 1998 to September 2019. Past performance is no guarantee of future results. Please see the end for important legal disclosures.

It is worth noting that some of these average correlations appear counterintuitive. For example, Low Volatility and Size factor premia exhibit a positive average correlation, despite their characteristic Z-scores being negatively correlated. This highlights the difference between regression-derived factor returns and those derived from long/short portfolios constructed by sorting stocks by their characteristic Z-scores. The regression approach considers all factor exposures, including industry and country effects, simultaneously, meaning only the return attributable to the factor itself is represented in the factor premium.

In contrast, characteristic Z-scores are not renormalized within countries or industries or with respect to the other factors' Z-scores. Hence, a pairwise correlation between factor Z-scores may represent multiple different relationships. As such, long/short portfolios constructed by sorting stocks by their characteristic Z-scores may have significant off-target exposures and exhibit returns that are not truly representative of the factor of interest.

Figures 8-11 plot the factor return correlation between Value and each remaining factor against the equivalent figure from the previous year (results for the additional pairs can be found in appendix B). The plots tend to cluster in the top-right and bottom-left quadrants of the graphs, suggesting some persistence in the correlation between factor returns. However, stability is less evident than in the case of volatility.

**Figures 8-11: Factor return correlations: Value**

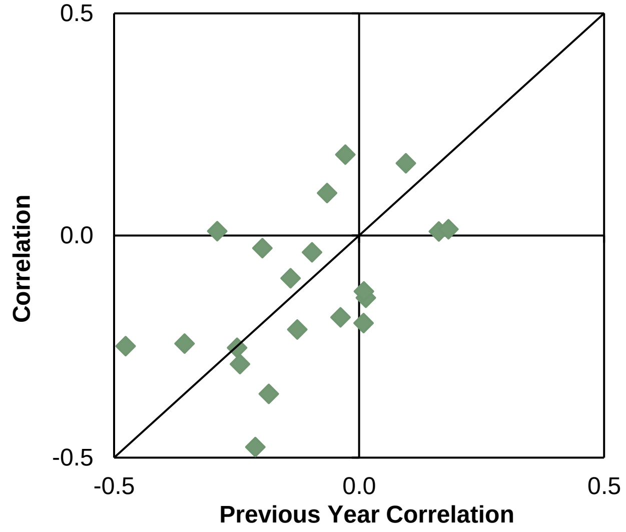


Figure 8: Value/Quality

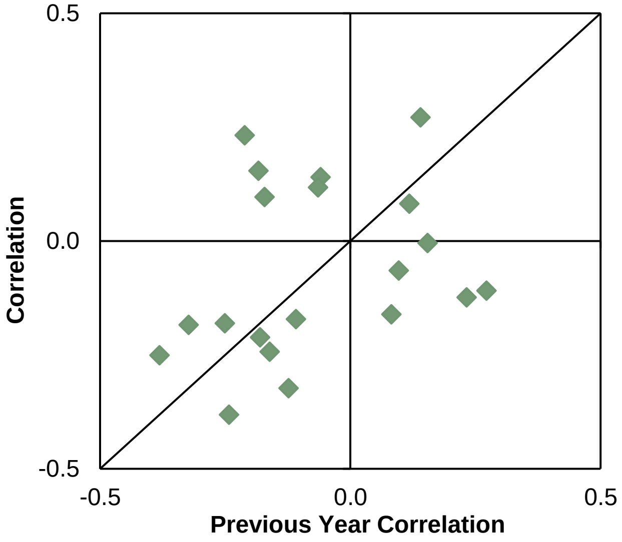


Figure 9: Value/Low Volatility

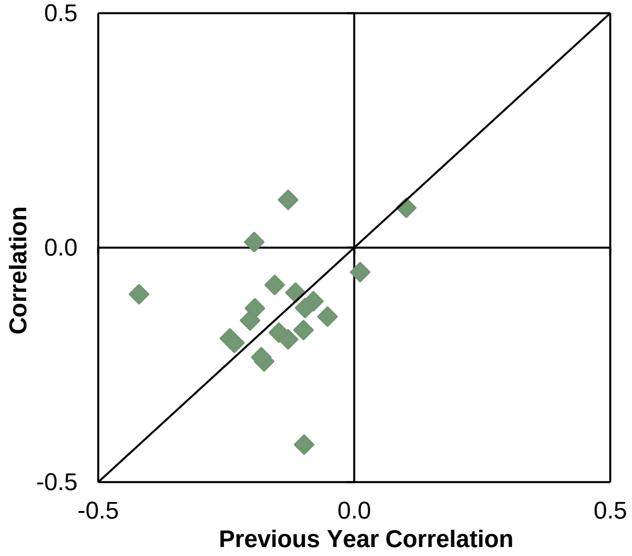


Figure 10: Value/Size

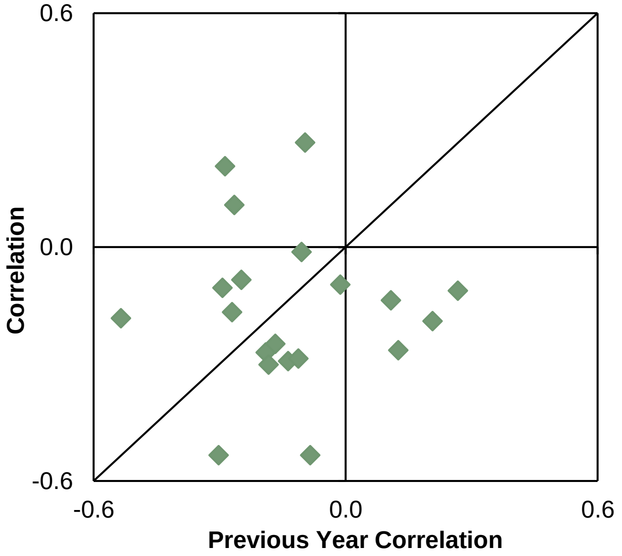


Figure 11: Value/Momentum

Source: FTSE Russell. Data based on the FTSE Developed Universe from August 1998 to September 2019. Past performance is no guarantee of future results. Please see the end for important legal disclosures.

We have observed that factor return correlations can be materially different from zero, suggesting that an ERC factor allocation scheme, which considers correlation in addition to volatility, may be required to achieve equality of factor contributions to active risk. However, these correlations are also somewhat unstable and may not be particularly useful as a predictor of future correlations.

There are a few key observations that arise from the results in this section:

- Factor volatility is not equal across factors
- Factor volatility (if scaled by market volatility) is a reasonable proxy for future factor volatility
- Correlations between factors can be significant
- There is some persistence in factor return correlations, but historic factor correlation is less representative of future correlation than is the case for volatility

# 3 Portfolio construction

We implement each of the three factor allocation schemes using a bottom-up portfolio construction approach. We use the FTSE Russell tilt framework, which provides a transparent mechanism for exercising precise control over single and multiple factor exposure objectives.

Equation (2) illustrates the basic concept of tilting; the application of a factor tilt to an initial set of portfolio weights results in a new portfolio with increased exposure to the factor. Increasing the strength of the tilt increases the level of factor exposure.

$$W_F = W_{Mcap} \times e^{n * Z_{Val}} \tag{2}$$

where $W_F$ represents the set of factor portfolio weights (one for each stock), $W_{Mcap}$ is an initial set of market capitalization weights, $e^x$ is the exponential function, $n$ is the strength of the tilt and $Z_{Val}$ is a set of factor Z-scores (in this case Value). The Z-scores result from the cross-sectional, capitalization-weighted normalization of a set of stock level characteristics. Equation (2) results in a set of unadjusted stock weights that require normalization or rescaling, such that they sum to one. We use this convention throughout this paper, where it is understood that the normalization is implicit.

The idea of the tilt is to create a portfolio that has increased exposure to the Value factor relative to the market capitalization-weighted portfolio. Portfolio level factor exposure is defined as the sum product of the set of portfolio weights $W$ and factor Z-scores $Z$:

$$\text{Exposure} = W \cdot Z \tag{3}$$

This is a point in time expression for the exposure of *any* portfolio $W$.

Equation (2) may be extended to incorporate multiple factors and to include country ($C$), industry ($I$) and capacity ($X$) tilts:

$$W_F = W_{Mcap} \times e^{n * Z_{Val}} \times e^{p * Z_{Qual}} \times e^{q * Z_{Vol}} \times e^{r * Z_{Size}} \times e^{s * Z_{Mom}} \times C \times I \times X \tag{4}$$

A set of tilt strengths ($n, p, q, r, s$) and country, industry and capacity tilts can be found by solving a system of equations, for each set of factor exposure, country, industry and capacity objectives. Using this approach, indexes can be constructed with a wide range of objectives, including “pure” factor indexes, which have active exposure to a desired factor, and no active exposure to any other factors, countries or industries. Here however, we utilize this approach to create multi-factor portfolios with the required exposure outcomes to implement the three factor allocation schemes of interest. For further details of the construction approach see [7].

Note that a top-down approach could be employed to create portfolios to implement each factor allocation scheme. In a top-down approach, the factor allocation objective is achieved through a composite or blending of independent “single factor” portfolio sleeves. The individual sleeves may then be weighted to achieve the desired factor allocation objective. However, a top-down factor allocation scheme will only be effective if the individual factor sleeves are relatively pure, i.e., they exhibit minimal off-target factor exposures. If material off-target factor exposures exist in the sleeves, then the aggregate exposures will not achieve the desired allocation as a result of interactions between the sleeves.

An accurate top-down factor allocation, however, may be implemented using the “pure” single factor indexes described above. However, as we have shown previously in [5], this typically results in lower levels of exposure than can be achieved through an equivalent bottom-up implementation.

# 4 Empirical results

We now examine the performance of the three factor exposure allocation schemes for the FTSE Developed universe of stocks over the period September 2000 to September 2019 and assess how closely the outcomes of each resemble a set of risk parity outcomes. Note the first two years of our data sample are used to form the initial factor covariance matrix, resulting in a sample period between September 1998 and September 2019.

We determine the factor allocation of each scheme semi-annually in March and September. A tracking error target is related to levels of factor exposure $E_i$ by:

$$[\text{Tracking Error Target}]^2 = \sum_{i=1}^{5}\sum_{j=1}^{5} E_i E_j C_{ij} = \boldsymbol{E^T C\, E} \tag{5}$$

where $C_{ij}$ is the covariance between the returns to factors $i$ and $j$ calculated using the methodology outlined in Section 2. The total active factor exposure will be split between each factor in accordance with each factor allocation scheme.

Note that equation (5) contains no estimate of residual risk. In the simulations that follow, we target a 2% tracking error, using a figure of approximately 1.8% in equation (5). In this way, we estimate that some 10% of active risk is not explained by our factors.

Solving equation (5) for an equal exposure objective, EE, is simple as we write $E_i = k$ for each factor $i$. For RE, we have $E_i = k/v_i$ where $v_i$ represents the $i^{th}$ factor return volatility. Finally, we solve the following equation for ERC:

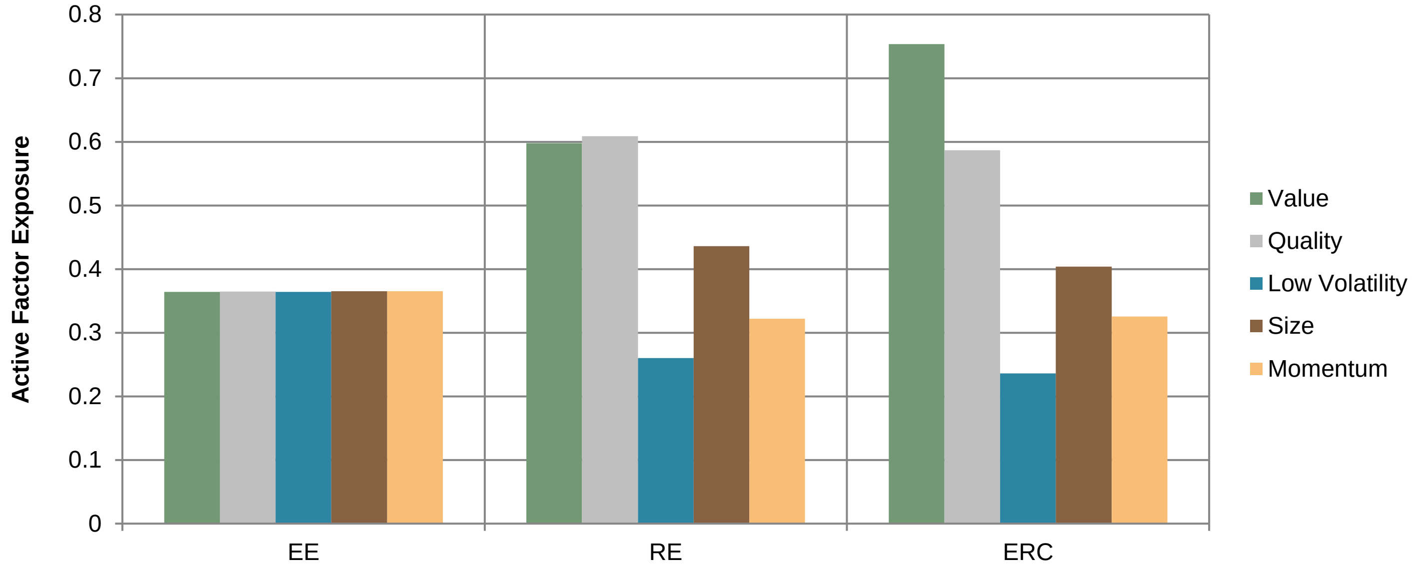
$$E_i(\boldsymbol{C\, E})_i = k \qquad \text{for each} \quad i \tag{6}$$

In each case, the solution fixes the ratios between factor exposures and we then set the magnitude of the scalar $k$ to achieve the tracking error target. The tilt strengths and the resulting portfolio weightings required to achieve these factor exposure objectives are determined by solving the system of equations described in section 3. The industry and country tilts are chosen so that the resulting portfolio has the same industry and country weights as the underlying market capitalization-weighted benchmark. The capacity tilt ensures that no stock's weight is greater than the minimum of 5% and 20 times the stock's market-capitalization weight.

## 4.1 Factor mix

Figure 12 displays the time-averaged active factor exposures of each allocation scheme. The RE and ERC portfolios display a similar exposure profile, which differs substantially from those that arise under EE. Broadly, as we move away from EE (equal exposure by construction), we see increased differences in exposure levels, suggesting that differences in factor volatilities are meaningful and are the primary determinants of both the RE and the ERC factor allocations. The level of Value exposure is the main difference between the RE and ERC schemes; Value exposure is higher under ERC because of its predominantly negative correlation with the other factors in the sample period.

**Figure 12: Average active factor exposure: factor allocation schemes**



Source: FTSE Russell. Data based on the FTSE Developed Universe from September 2000 to September 2019. Past performance is no guarantee of future results. EE, RE, and ERC indexes been created for research purposes only. Please see the end for important legal disclosures.

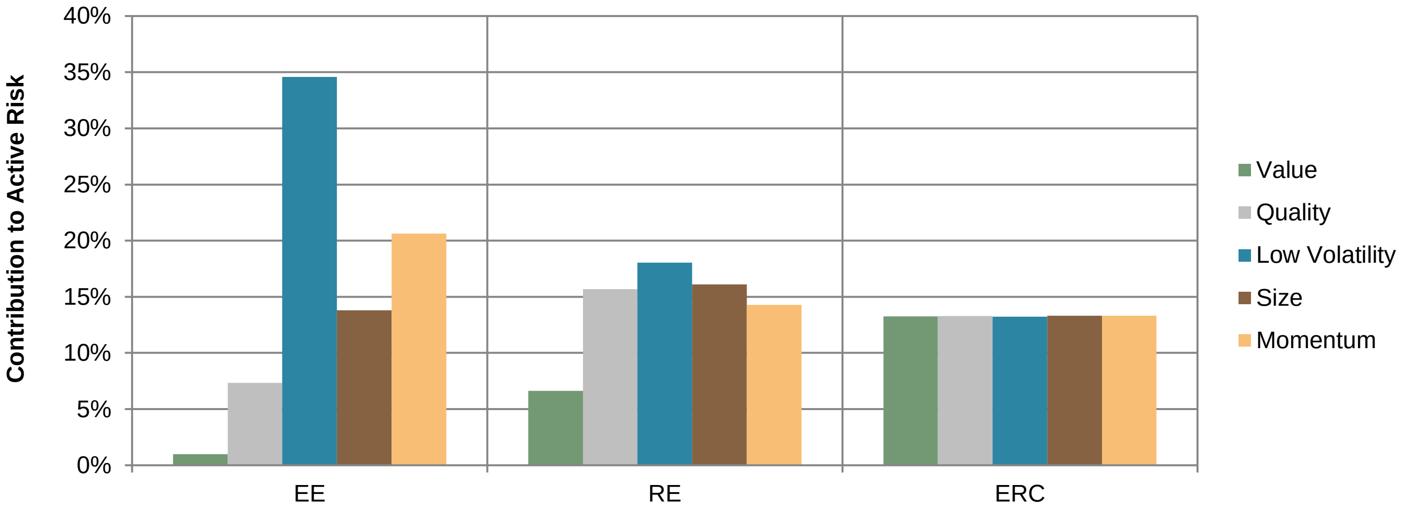
It is worth observing that had a top-down approach been employed to create these portfolios using a set of pure single factor building blocks with unit target exposure, the maximum exposure achievable in the EE portfolio would have been 0.2 units per factor. This corresponds to a tracking error of roughly 1%. In other words, a tracking error target of 2% is difficult to achieve, even when using relatively high exposure single factor portfolio sleeves. Similarly, RE and ERC portfolios with 2% tracking error targets would prove difficult to achieve in a top-down manner, emphasizing the limitations of top-down approaches.

## 4.2 Active risk attribution

Figure 13 shows the average ex-ante active risk contribution from each factor for each allocation scheme. These figures are average active risk attributions determined at each semi-annual rebalance over the simulation period. Each attribution uses return data for the two years preceding each rebalance.

The ex-ante active risk contribution of each factor under ERC is equal by construction; RE results in ex-ante active risk contributions that are close to parity, while active risk contributions under EE are far from equal. This is in contrast to the pattern of factor exposures of the EE mechanism and the increasingly unequal exposures of the RE and ERC approaches. Each approach is industry and country neutral, with respect to the underlying benchmark and hence the contribution to active risk from these sources is minimal. The contributions from stock specific risk increases as we move from EE (23%) to RE (29%) and ERC (34%), as portfolios become increasingly concentrated. We expand upon this in section 4.3.

**Figure 13: Average ex-ante active risk contribution: factor allocation schemes**

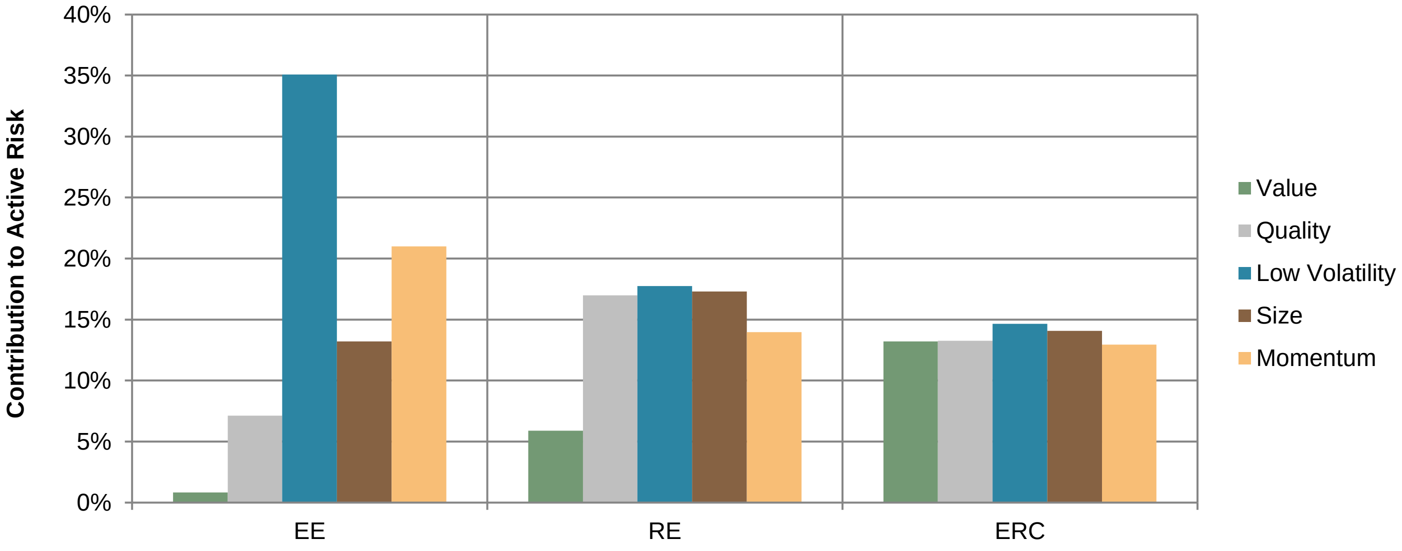


Source: FTSE Russell. Data based on the FTSE Developed Universe from September 2000 to September 2019. Past performance is no guarantee of future results. EE, RE, and ERC indexes been created for research purposes only. Please see the end for important legal disclosures.

Figure 14 shows the average ex-post active risk contributions. These figures are calculated in the same way as the ex-ante figures, but use realized factor return data from the two years following each rebalance rather than from the two years preceding each rebalance.

Ex-post active risk contributions are similar to the ex-ante contributions, suggesting that despite variability in factor correlations, an ERC weighting scheme results in close to equal ex-post active risk contributions. Hence, such a factor allocation regime can effectively be implemented using historical volatilities and correlations.

**Figure 14: Average ex-post active risk contribution: factor allocation schemes**

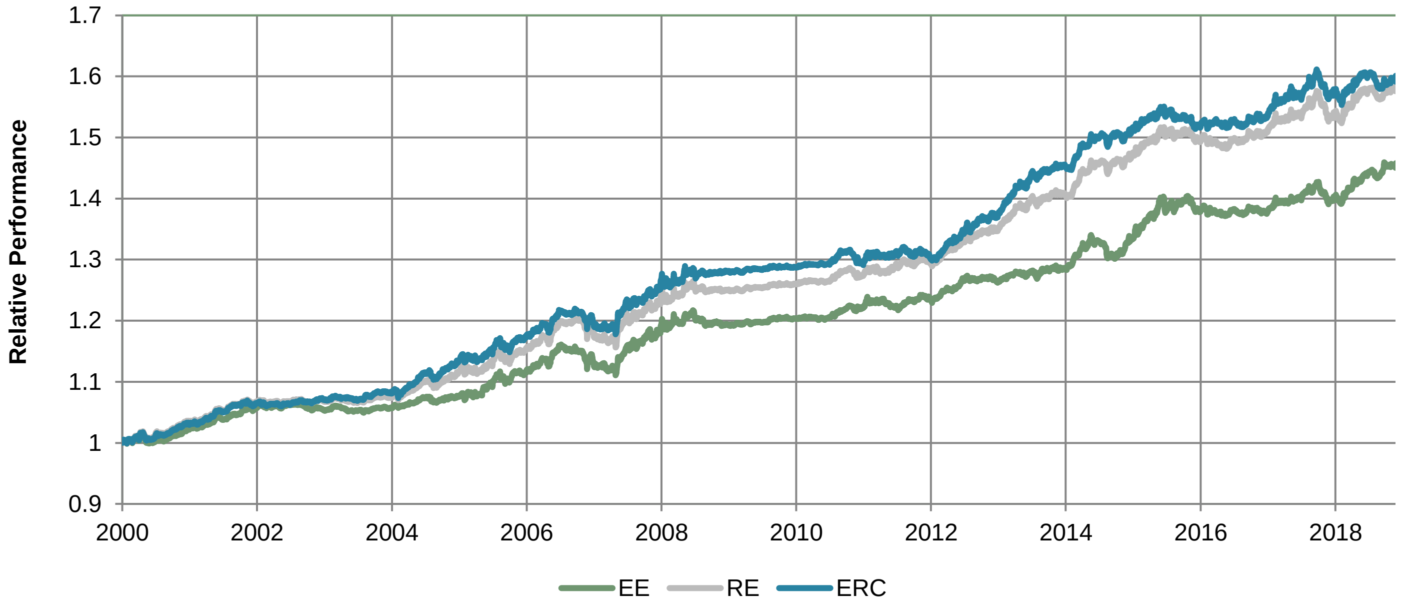


Source: FTSE Russell. Data based on the FTSE Developed Universe from September 2000 to September 2019. Past performance is no guarantee of future results. EE, RE, and ERC indexes been created for research purposes only. Please see the end for important legal disclosures.

## 4.3 Performance

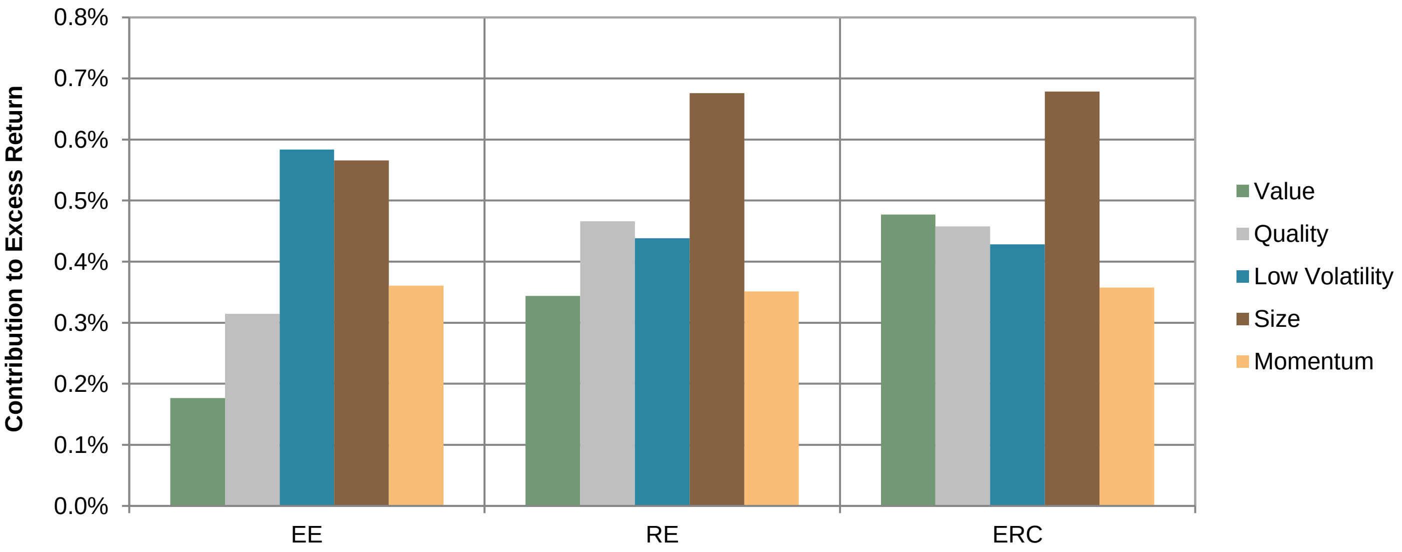
Figure 15 shows the relative performance of the three allocation schemes versus the FTSE Developed Index, and Table 2 the key performance metrics. The RE & ERC allocation schemes display very similar performance outcomes and deviate from those of EE. Figure 16 presents a performance attribution for each allocation scheme. This indicates that the primary cause of the performance differential is the higher exposures to (and therefore contributions from) Value, Quality and Size under the RE & ERC schemes. This more than compensates for their lower exposure to Low Volatility.

**Figure 15: Performance relative to FTSE Developed: factor allocation schemes**



Source: FTSE Russell. Data based on the FTSE Developed Universe from September 2000 to September 2019. Past performance is no guarantee of future results. EE, RE, and ERC indexes been created for research purposes only. Please see the end for important legal disclosures.

**Figure 16: Performance attribution (average excess return per annum): factor allocation schemes**



Source: FTSE Russell. Data based on the FTSE Developed Universe at rebalance from September 2000 to September 2019. Past performance is no guarantee of future results. EE, RE, and ERC indexes been created for research purposes only. Please see the end for important legal disclosures.

Interestingly, the portfolios become more concentrated as one moves from EE to RE to ERC. This is illustrated by the decreasing Effective N in Table 2. The increase in stock specific risk alluded to earlier has its roots in this. Turnover also gradually rises as we move from EE to RE to ERC. This is explained by the fact that the relative exposure targets for EE do not move through time while those for RE and ERC will vary along with factor volatility and correlation.

**Table 2: Performance, implementation and diversification: factor allocation schemes**

| | FTSE Developed | EE | RE | ERC |
|---|---|---|---|---|
| Geo. Return (% p.a.) | 5.25 | 7.33 | 7.79 | 7.85 |
| Volatility (% p.a.) | 15.75 | 15.59 | 15.68 | 15.73 |
| Sharpe Ratio | 0.33 | 0.47 | 0.50 | 0.50 |
| Volatility Reduction (%) | | 1.02 | 0.44 | 0.16 |
| Max DD (%) | -57.37 | -53.93 | -54.07 | -53.88 |
| 2-Way Turnover (% p.a.) | 13.03 | 142.93 | 155.79 | 159.75 |
| | | | | |
| Excess Return (%p.a.) | | 2.08 | 2.54 | 2.60 |
| Tracking Error (%p.a.) | | 1.89 | 1.99 | 2.06 |
| Information Ratio | | 1.10 | 1.27 | 1.26 |
| | | | | |
| Effective N | 350 | 223 | 200 | 179 |

Source: FTSE Russell. Data based on the FTSE Developed Universe at rebalance from September 2000 to September 2019. Past performance is no guarantee of future results. EE, RE, and ERC indexes been created for research purposes only. Please see the end for important legal disclosures.

Table 2 also shows that the three indices display very similar realized tracking errors that are consistent with the ex-ante target of 2%p.a. This is consistent with the ex-post ERC risk contribution figures shown earlier and illustrates that there is value in the use of the historical factor covariance matrix to inform forward looking factor allocation decisions.

# 5 Conclusions

As the popularity of factor investing increases, we believe that the factor allocation decision will become a more prominent consideration for investors. By this, we mean that it will encompass more than the mere identification of a set of desirable factors, but extend to cover the explicit choice over the relative magnitude of each factor's exposure in conjunction with a tracking error budget. In this paper we have presented three common allocation schemes, which we hope will aide investors seeking balanced exposure to multiple factors. In addition to the three specific allocation schemes, we have presented a transparent, non-optimized, bottom-up portfolio construction mechanism applicable to the implementation of a broad range of factor objectives.

Each of the three allocation schemes examined delivers similar absolute risk-adjusted performance outcomes; the Risk Exposure and Equal Risk Contribution schemes display a small uplift in information ratios compared to the Equal Exposure scheme. In the absence of explicit beliefs regarding the future payoff of each factor, a sensible objective is to ensure that factor exposure is suitably diversified. We have seen that factor return volatilities and correlations matter since the Equal Exposure factor mix results in neither equal ex-ante nor ex-post contributions to active risk. In contrast, while the factor exposures of the Risk Exposure and Equal Risk Contribution mechanism are decidedly unequal, both result in ex-ante and ex-post active risk contributions that are approximately equal.

An Equal Exposure approach to factor allocation is unlikely to achieve factor risk parity outcomes so long as the volatility of factor risk premia differ; it will overweight high volatility factors, such as Low Volatility and Momentum, and underweight less volatile factor attributes, such as Quality and Value. Hence, an Equal Exposure factor allocation scheme will potentially deliver sub-optimal levels of factor diversification. In contrast, a Risk Exposure approach appears to provide reasonably balanced risk contribution outcomes. However, to achieve true parity of risk contributions, factor correlations are a necessary consideration, which are employed in an Equal Risk Contribution approach.

As demonstrated in the sample period in this paper, relatively simple historic measures of factor volatility and correlation provided reasonable estimates of their future values and may be practically employed in a framework for allocating to factors within a tracking error budget. This suggests that in addition to providing a formal means of approaching the factor allocation decision, such an approach may also have merit for the creation of practical and meaningful generic benchmarks for the evaluation of a wide range of multi-factor portfolios.

# 6 Appendix A

This appendix contains the definitions of the metrics used in this document.

## 6.1 Z-scores

We define a set of factor characteristics $f_i$ for each stock labeled by $i = 1, \ldots, N$ where $N$ is the number of stocks in our universe. Let $\omega_i$ be any set of positive weights which sum to one, then the weighted Z-score is defined by:

$$Z_{\omega\ i} = (f_i - \mu_\omega)\ /\sigma_\omega \tag{2}$$

where the weighted mean and variance are:

$$\mu_\omega = \sum_{i=1}^{N} \omega_i * f_i \qquad \text{and} \qquad {\sigma_\omega}^2 = \sum_{i=1}^{N} \omega_i * (f_i - \mu_\omega)^{\ 2} \tag{3}$$

The set of weights $\omega_i$ may take any form, but in this paper we employ either a set of equal weights defined by $E_i = 1/N$ or market capitalization weights:

$$W_{M,i} = \frac{M_i}{\sum_{i=1}^{N} M_i} \tag{4}$$

where $M_i$ is the free-float adjusted market capitalization of the $i^{th}$ stock. When equal weights are employed to calculate the weighted mean, standard deviation and Z-Score, they reduce to the usual un-weighted expressions.

## 6.2 Exposure

To assess how much of a characteristic is embedded in a portfolio at a given point in time, we define exposure as:

$$\text{Exposure} = W \cdot Z = \sum_{i=1}^{N} W_i * Z_i \tag{5}$$

where $W$ is a set of portfolio weights and $Z$ is as set of Z-scores. The active exposure, relative to set of market capitalization weights $W_M$ is defined by:

$$\text{Active Exposure} = W \cdot Z - W_M \cdot Z \tag{6}$$

## 6.3 Diversification

To assess the degree of diversification in portfolio, we define Effective N of a portfolio as the inverse of the Herfindahl measure of concentration [6]:

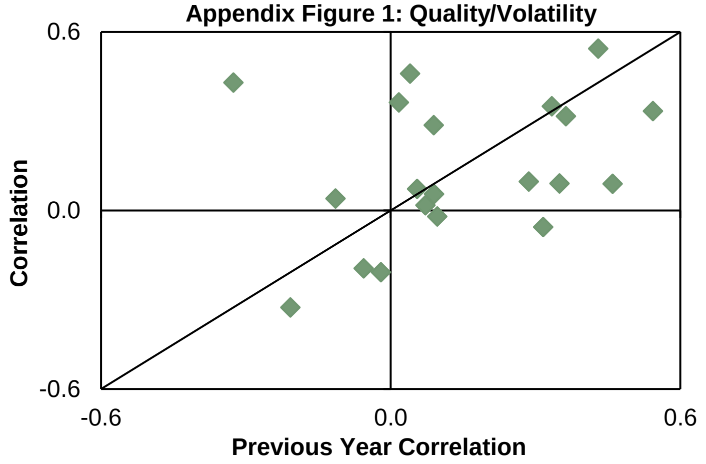
$$\text{Effective N} = 1/(W \cdot W) = 1/\sum_{i=1}^{N} {W_i}^2 \tag{7}$$

Effective N attains its maximum under an equal weighting scheme when it is equal to the actual number of stocks. Hence, Effective N can be seen as a measure of “how far” a given portfolio is from this maximally diversified portfolio.

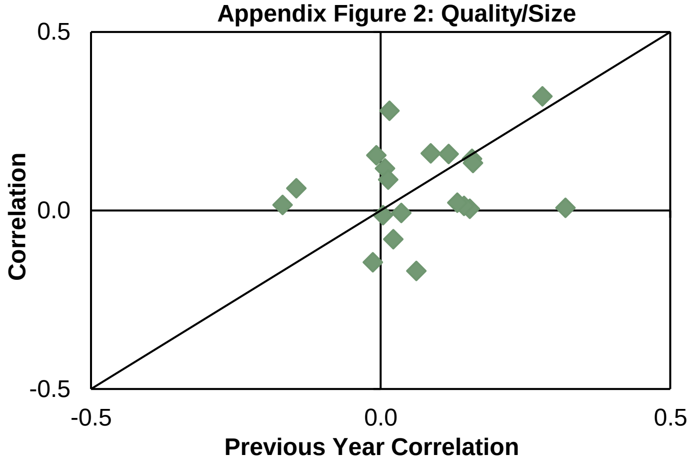
# 7 Appendix B

This appendix contains the correlation stability results for the six remaining factor pairs for the FTSE Developed universe.

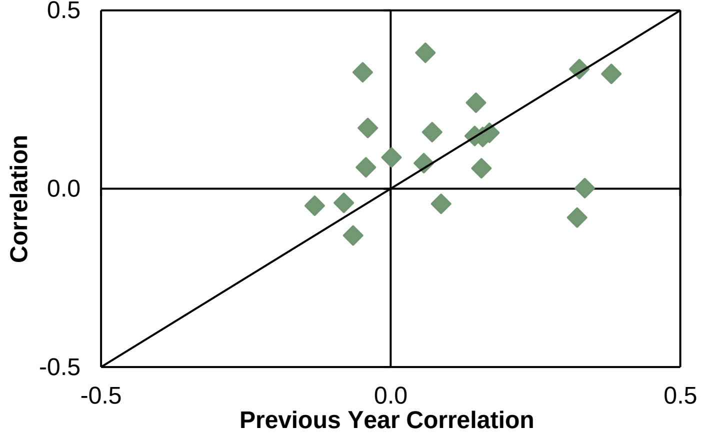
### Appendix Figures 1-6: Factor Return Correlations



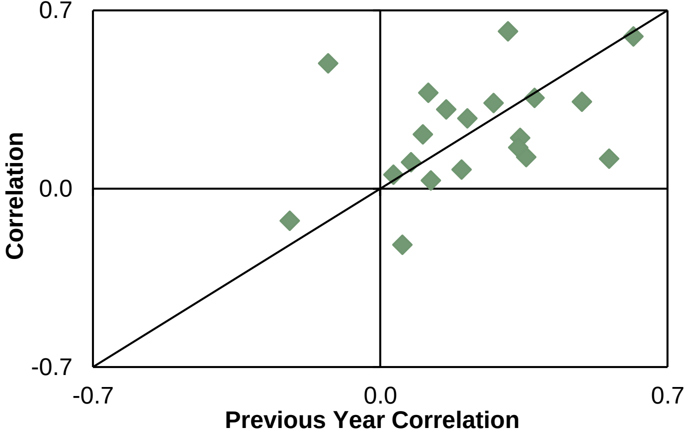
Appendix Figure 1: Quality/Volatility



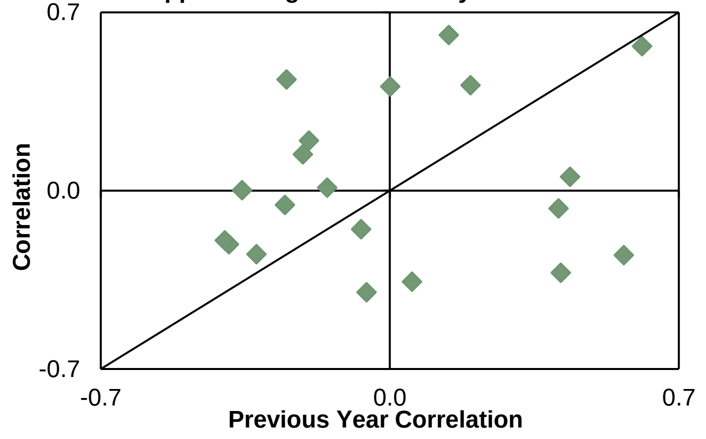
Appendix Figure 2: Quality/Size



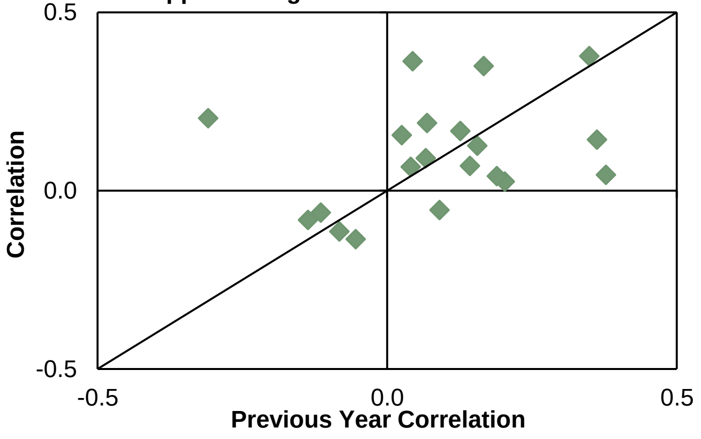
Appendix Figure 3: Quality/Momentum



Appendix Figure 4: Volatility/Size



Appendix Figure 5: Volatility/Momentum



Appendix Figure 6: Size/Momentum

Source: FTSE Russell. Data based on the FTSE Developed Universe from August 1998 to September 2019. Past performance is no guarantee of future results. Please see the end for important legal disclosures.

# 8 References

[1] “Factor behavior through the cycle”, FTSE Russell, June 2019.

[2] Plyakha, Y., Uppal, R., Vilkov, G., “Equal or Value Weighting? Implications for Asset-Pricing Tests”, EDHEC White Paper, April 2014.

[3] Hurst, B., Johnson, B., Hua Ooi, Y., “Understanding Risk Parity”, AQR White Paper, February 2010.

[4] Maillard, S., Roncalli, T., Teiletche, J., “On the Properties of Equally-Weighted Risk Contribution Portfolios”, *Journal of Portfolio Management*, 36 (4) 60-70, Summer 2010.

[5] “Top-down or bottom-up?”, FTSE Russell, March 2018.

[6] Woerheide, W. and Persson, D. “An Index of Portfolio Diversification”, *Financial Services Review*, Vol. 2, p73-p85, 1993.

[7] FTSE Global Factor Index Series Ground Rules, FTSE Russell, February 2020.

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