Ground Rules

FTSE Russell

Guide to Calculation Methods for the FTSE Fixed Income Indices

v1.9

Contents

Section 1

Introduction

1.0 Introduction

1.1 The aims of the guide are:

A. To describe how FTSE Fixed Income indices are calculated;

B. To make it easier for users to replicate the indices in order to support their investment and trading activities; and

C. To assist users in understanding the components which influence the performance of the indices.

1.2 The guide is set out into two further sections, section 2 covers index level calculations such as index total return and index yield for example. Section 3 covers bond level calculations such as accrued interested, bond yield and duration.

1.3 FTSE Russell is a trading name of FTSE International Limited, Frank Russell Company, FTSE Global Debt Capital Markets Limited (and its subsidiaries FTSE Global Debt Capital Markets Inc. and FTSE Fixed Income Europe Limited), FTSE Fixed Income LLC, The Yield Book Inc and Beyond Ratings.

Section 2

Index level calculations

2.0 Index level calculations

This section details the common index level calculations that are used across different FTSE fixed income index families, which refer to this guide. For each index calculation, a description is given along with the formulae, and a list of the index families that the calculation applies to.

The key to the notation used in this document can be found in Appendix 1.

2.1 Total Return Index (TRI)

The total return index seeks to replicate the return from holding the index portfolio; it gives the market value weighted return of the index constituents, taking into account price movements, accrued interest and cash-flows from the bonds (including coupon payments, redemptions or repurchases).

The generalised total return index can be expressed as:

$$TR_t = TR_{t-1} \times \frac{\sum_{i=1}^{n}\left(\left(\left[P_{i,t} + AI_{i,t} + XDIV_{i,t}\right]N_{i,t-1}.R_{i,t}.CF_{i,t} + Cash_{i,t}\right).FX_{i,t}\right)}{\sum_{i=1}^{n}\left(\left(\left[P_{i,t-1} + AI_{i,t-1} + XDIV_{i,t-1}\right]N_{i,t-1}.R_{i,t-1}.CF_{i,t} + Cash_{i,t-1}\right).FX_{i,t-1}\right)}$$

For indices that do not have bonds that go ex-dividend or do not contain capping, the formula can be simplified:

$$TR_t = TR_{t-1} \times \frac{\sum_{i=1}^{n}\left(\left(\left[P_{i,t} + AI_{i,t}\right]N_{i,t-1}.R_{i,t} + Cash_{i,t}\right).FX_{i,t}\right)}{\sum_{i=1}^{n}\left(\left(\left[P_{i,t-1} + AI_{i,t-1}\right]N_{i,t-1}.R_{i,t-1} + Cash_{i,t-1}\right).FX_{i,t-1}\right)}$$

2.1.1 Index Cash

The cash term in the total return formula, Cash_(i,t)is the sum of any coupons, G_(i,t) and any principal repayments, RD_(i,t) from bond i, which have accumulated since the last cashflow re-investment date until the calculation date, t and can be expressed as:

$$Cash_{i,t} = Cash_{i,t-1} + \left(G_{i,t} + RD_{i,t}\right)$$

where:

$$G_{i,t} = \left(\frac{c_i}{f_i}\right) XD\_Marker_i.N_{i,t-1}.R_{i,c}.CF_{i,t}$$

and:

$$RD_{i,t}=\left[\left(R_{i,t-1}-R_{i,t}\right)RP_{i,t}\right].N_{i,t-1}\ CF_{i,t}$$

For indices that re-invest cash on a daily basis, Cash_(i,t-1), is always zero. For indices that re-invest cash on a monthly basis, Cash_(i,t-1) is the cashflow assigned to the bond on the previous calculation date, unless t-1 is the cash re-investment date, in which case it is set to zero.

2.1.2 Ex-Dividend treatment

For bonds that go ex-dividend, bondholders are not eligible to receive the next coupon if they are not the holder of the bond prior to the ex-dividend date. This has two implications on the total return, which are accounted for the total return index formula in section 2.1.1

1. **Accrued Interest** – When a bond is trading ex-dividend the accrued interest turns negative. When chain-linking with the previous calculation day's accrued interest, an adjustment needs to be made to reflect this drop. This is done via $XDIV_{(i,t)}$:

$$XDIV_{i,t}=\left(\frac{c_i}{f_i}\right)XD\_Marker_i.N_{i,t}.R_{i,c}\ .CF_{i,t}$$

For bonds which do not have coupons that go ex-dividend XDIV_(i,t)=0.

2. **Index Rebalancing.** If a bond enters an index during its ex-dividend period, the next coupon payment will not be recognised in the total return as it is assumed that anyone tracking the index will have bought the bond during its ex-dividend period, and will not be eligible to receive the next coupon payment. If a bond does enter the index during its ex-dividend period, X〖D_Marker〗$_i$=0, otherwise it is X〖D_Marker〗$_i$=1.

2.1.3 Capping Factors

The total return formula in section 2.1.1 takes into account any capping that is applicable to bond i.

$$CF_{i,t}=\frac{\text{Capped Bond Notional}_i}{N_{i,t}}$$

If the index does not include any capping, or the bond is not capped, the Capping Factor, CF_iis equal to 1. The capping factor is reset on any date that the index capping is applied, which is usually the same day as the index is rebalanced.

2.1.4 **Principal Redemptions**

Any partial or complete redemptions need to be accounted for in the calculation of the total return, so that i) any redemption income is realised and ii) the market value of the remaining bond is adjusted downward, proportional to the amount that is redeemed. In the total return index formula in section 2.1.1, this is handled by the redemption factor, R_(i,t).

$$R_{i,t}=\frac{\text{New Bond Outstanding Amount}}{N_{i,t}}$$

As $N_{(i,t)}$ is reset to the current bond outstanding amount on each rebalancing date, $R_{(i,t)}$ is reset to equal 1 on each rebalancing date.

2.2 Clean Price Index (PRI)

2.2.1 **PRI: Standard Formula**

The standard clean price index takes into account the market value weighted clean-price movements of the index constituents:

$$PR_t=PR_{t-1}\times\frac{\sum_{i=1}^{n}\left(P_{i,t}N_{i,t-1.}R_{i,t}.CF_{i,t.}.FX_{i,t}\right)}{\sum_{i=1}^{n}\left(P_{i,t-1}N_{i,t-1.}R_{i,t}.CF_{i,t.}.FX_{i,t}\right)}$$

2.3 Gross Price Index (PRI)

2.4 Index Yield

2.4.1 **Index Yield: Duration\MVW Weighting**

The standard approach to calculate index yield is to weight the average yield of the constituents by modified duration and market value (MVW):

$$\text{Index Yield}_t=\frac{\sum_{i=1}^{n}\left(MV_{i,t}.Y_{i,t.}MD_{i,t}.FX_{i,t}\right)}{\sum_{i=1}^{n}\left(MV_{i,t}.MD_{i,t}.FX_{i,t}\right)}$$

Where the market value of bond i on date t, $MV_{(i,t)}$, is given by

$$MV_{i,t}=\left(P_{i,t}+A_{i,t}\right)N_{i,t-1,}R_{i,t}.CF_{i,t}$$

2.5 Index Rate Sensitivities

2.5.1 **Index Duration and Convexity: MVW**

The Index Duration, Modified Duration, DV01(if applicable) and Convexity are weighted market value:

$$\text{Index\_Sensitivity}_t=\frac{\sum_{i=1}^{n}\left(MV_{i,t}.\text{Sensitivity}_{i,t.}.FX_{i,t}\right)}{\sum_{i=1}^{n}\left(MV_{i,t}.FX_{i,t}\right)}$$

Where $\text{Sensitivity}_{(i,t.)}$ is the duration, DV01, modified duration or convexity of bond i on date t.

2.6 Average Coupon

2.6.1 **Index Average Coupon**

The average coupon level is the notional weighted average coupon rate:

$$\text{Average Coupon}_t=\frac{\sum_{i=1}^{n}\left(c_{i,t.}N_{i,t.}R_{i,t}.CF_{i,t}.FX_{i,t}\right)}{\sum_{i=1}^{n}\left(N_{i,t.}R_{i,t}.CF_{i,t}.FX_{i,t}\right)}$$

2.7 Average Remaining Life

2.7.1 **Index Average Remaining Life (MVW)**

This measure gives the weighted average remaining life of all the constituents in the index.

$$w_{i,t} = \frac{N_{i,t.} R_{i,t}.CF_{i,t}.FX_{i,t}}{\sum_{i=1}^{n}\left(N_{i,t.} R_{i,t}.CF_{i,t}.FX_{i,t}\right)}$$

Section 3

Bond level calculations

3.0 Bond level calculations

3.1 Accrued Interest

The accrued interest is calculated as the interest payable when purchasing a bond, this is to account for the interest income that is due to the previous owner of the bond, who will not be receiving the next coupon payment. Accrued interest is calculated as follows:

$$A_{i,t} = \text{fractional interest period}_{i,t} \times \frac{C_i}{f_i}$$

For bonds whose coupons do not go ex-dividend, or are not in their ex-dividend period, *fractional interest period*$_{(i,t)}$ is calculated as the time between the previous coupon date and the settlement date, in terms of coupon periods, according to the day count convention of the bond. More specifically, the *fractional interest period*$_{(i,t)}$ is:

$$\text{fractional interest period}_{i,t} = \frac{\text{Settlement Date} - \text{Previous Coupon Date}}{\text{Number of days in coupon period}}$$

The number of days in the numerator and the denominator are calculated according to the day count convention of the bond. For further details on day count conventions, please consult Appendix 2.

For bonds which are currently in the ex-dividend coupon period, the fractional coupon period is calculated as the time between the next coupon date and the settlement date, in terms of coupon periods, according to the day count convention of the bond. Since the next coupon date is always after the settlement date, the fractional coupon period is negative. More specifically, thefractional interest period_(i,t) for bonds in their ex-dividend period is:

$$\text{fractional interest period}_{i,t} = \frac{\text{Next Coupon Date} - \text{Settlement Date}}{\text{Number of days in coupon period}}$$

The calculation of accrued interest, accounts for odd first and last coupons.

3.2 Yield to Maturity

The yield to maturity of a bond is the constant discount rate which will discount all future cashflows of a bond to equal its current clean price plus accrued interest (dirty price). The yield to maturity is found by solving the following formula iteratively:

$$P_{i,t}+A_{i,t}=\sum_{k=\frac{v}{r}}^{m+\frac{v}{r}}\left[CashFlow_{i,k}.\frac{1}{\left(1+\frac{Y_{i,t}}{f_i}\right)^k}\right]$$

The compounding frequency of the calculated yield to maturity, $Y_{(i,t)}$ corresponds to the compounding frequency of the bond. For example, if the coupon frequency is semi-annual (f_i=2), then the calculated yield will be compounded on a semi-annual basis.

3.2.1 Yield Conversions

In order to annualise a semi-annually compounded yield, the following formula is used:

$$annual\ yield=\left(1+\frac{Y_{i,t}}{2}\right)^2-1$$

In order to derive a semi-annually compounded yield from an annual yield, the following formula is used:

$$semi\_annual\ yield=2\sqrt{Y_{i,t}+1}-2$$

3.2.2 Simple Yield

For bonds entering their final coupon period, the simple yield is calculated, on a money market basis (ACT/360 or ACT/365) instead of yield to maturity.

$$SY_{i,t}=\frac{cashflow_{i,k}-(P_{i,t}+A_{i,t})}{P_{i,t}+A_{i,t}}.\frac{360\text{ or }365}{v}$$

3.3 Macaulay Duration

The Macaulay duration of a bond is the time weighted average of the remaining cashflows. It can be calculated as:

$$MacDur_{i,t}=\frac{\sum_{k=\frac{v}{r}}^{M+\frac{v}{r}}\left[CashFlow_k.\frac{k}{\left(1+\frac{Y_{i,t}}{f_i}\right)^k}\right]}{\left(P_{i,t}+A_{i,t}\right).f_i}$$

3.4 Modified Duration

The modified duration of a bond is the percentage change in the price arising from a 1% change in the yield to maturity. It can be calculated as:

$$ModDur_{i,t}=\frac{MacDur_{i,t}}{\left(1+Y_{i,t}\right)}$$

3.5 Convexity

The convexity of a bond is the change in modified duration arising from a 1% change in the yield to maturity. It can be calculated as:

$$Convx_{i,t}=\frac{\sum_{k=\frac{v}{r}}^{M+\frac{v}{r}}\left[CashFlow_{k}\cdot\left(\frac{k}{f_i}+\frac{k^2}{f_i^2}\right)\cdot\frac{1}{\left(1+Y_{i,t}\right)^k}\right]}{\left(P_{i,t}+A_{i,t}\right)\left(1+Y_{i,t}\right)^2}$$

3.6 DV01

The DV01 measure shows the absolute change in the bond price for a 1bp change in the yield. It can be calculated as the product of the dirty (or gross) price of a bond and its modified duration:

$$DV01_{i,t}=(P_{i,t}+A_{i,t})\times ModDur_{i,t}$$

Appendix 1: Key to terms

Key to Terms	
$AI_{i,t}$	The accrued interest for bond i on date t
c_i	The coupon rate of bond i
$Capped\ Bond\ Notional_i$	The capped notional amount of bond I, this is fixed at the last index capping date.
$Cash_{i,t}$	The total cashflows from bond i on date t since the last rebalance date
$CashFlow_{i,k}$	The cashflow due from bond i at time period k
CF_i	The capping factor applicable to bond i
$Convx_{i,t}$	The convexity of bond i at time t
$DV01_{i,t}$	The dollar value of a basis point of bond i at time t
f_i	The number of coupon payments per year from bond i
$FX_{i,t}$	The foreign exchange rate associated with bond i at time t. It is quoted as local/base and in cases where base currency of the index is the same as the local currency, the rate is 1.
$G_{i,t}$	The coupon income received from bond i at time t
k	Counter to indicate the time in coupon periods
m	Superscript to denote the complete number of coupon periods until the maturity date
$MacDur_{i,t}$	The Macaulay duration of bond i at time t
$MD_{i,t}$	The modified duration of bond i at time t
$MV_{i,t}$	The market value of bond i at time t
n	Superscript to denote the number of bonds in the index
$N_{i,t}$	The nominal amount outstanding amount of bond i at time t. This is fixed at the last index rebalancing date.
$_t$and $_{t-1}$	Time subscripts to denote the current and previous calculation date.
$P_{i,t}$	The clean price of bond i on date t
$R_{i,t}$	Fraction to denote the cumulative amount of bond i that has been repurchased, up until date t, since the previous rebalancing date.
$RD_{i,t}$	The redemption income due from bond i on date t
$RP_{i,t}$	The redemption price of bond i at time t, if the bond is partially or entirely

	redeemed on date t.
r	The number of days in the coupon period, calculated according to the day count convention of the bond
$Sensitivity_{i,t.}$	Can represent $MacDur_{i,t}$, $MD_{i,t}$,$Convx_{i,t}$ or $DV01_{i,t}$
$SY_{i,t}$	The simple yield of bond i at time t
v	The number of dates between day i and the next cash flow date, calculated according to the day count convention of the bond
$XDIV_{i,t}$	This is the coupon that is payable by bond i on date t, during its ex-div period. Otherwise it is zero.
XD_Marker_i	This is set to 0 if a bond enters the index during its coupon ex-dividend date. If it enters the index outside its ex-dividend period, or does not go ex-dividend, it is set to 1.
$Y_{i,t.}$	The yield to maturity for bond i at time t

Appendix 2: Day to Count Conventions

Bond level calculations such as yield, accrued interest, duration and convexity rely on the day-count convention of each bond. These conventions describe the assumptions that are used when calculating the number of days between two dates.

The main day count conventions are:

- ACT/ACT
- ACT/365
- ACT/360
- 30/360
- 30/360 (US)
- 30/360 (Eurobond)

The first part of the convention name, before the "/", indicates the assumed number of days in a month between two dates. The part after the "/" indicates the number of assumed days in a year. When using the 30/360 convention for example, the "30" means that there are an assumed 30 days in each month between two dates, and the 360 means that there are 360 days in each year between two dates. ACT means that the actual number of days are counted.

The 30/360 (US) and the 30/360 (Euro) conventions are similar to the 30/360, but vary in the adjustments to the start and end dates:

30/360 (US)

1. If the day of the start date (D1) is 31, D1 is changed to 30, then
2. If the day of the end date (D2) is 31, and D1 is 30, then change D2 to 30.

30/360 (Euro)

1. If the day of the start date (D1) is 31, then D1 is changed to 30, then
2. If the day of the end date (D2) is 31, then change D2 to 30.

Additionally, there are conventions on the treatment of non-business days, such as:

- **Following** - If a date falls on a non-business day, it is moved forward to the next business day.
- **Modified Following** - If a date falls on a non-business day, it is moved forward to the next business day, unless that day is in the next calendar month, in which case the previous business day is used.
- **End of Month** - This means that each date is adjusted so that it falls at the on the last calendar day of the month.

In order to illustrate the different day count conventions, the examples below show how the accrued interest is calculated for a bond under various scenarios.

Example 1 – ACT/ACT

Coupon	2.75% semi-annual
Maturity	21 Apr 2024
Settlement Date	04 Aug 2014
Previous Coupon Date	21 Apr 2014
Next Coupon date	21 Oct 2014

In this example, the bond pays a coupon of 2.75 semi-annually, and uses the ACT/ACT day count convention. The accrued interest can be calculated as:

$$A_{i,t}=\text{fractional interest period}_{i,t} \times \frac{C_i}{f_i}$$

where:

$$\text{fractional interest period\_(i,t)}=\frac{\text{Settlement Date-Previous Coupon Date}}{\text{Number of days in coupon period}}$$

Using the ACT/ACT convention, the actual number of calendar days between the settlement date of 04 August 2014 and the previous coupon date of 21 Apr 2014 is 105 days.

The actual number of days in the coupon period, between the previous coupon date of 21 April 2014 and the next coupon date of 21 October 2014 is 183 days.

This gives the *fractional interest period*$_{i,t}$ as:

$$\text{fractional interest period}_{i,t} = \frac{105}{183} = 0.57377$$

The accrued interest is therefore:

$$A_{i,t} = 0.57377 \times \frac{2.75\%}{2} = 0.78893$$

Example 2 – ACT/365

Using the ACT/365convention, the actual number of days in a month are used, but each year is assumed to consist of 365 days. Using the example above, the numerator and the denominated of the fractional interest period$_{(i,t)}$ is calculated as follows:

Settlement date – Previous Coupon Date	=	04 August 2014 – 21 April 2014
	=	105 days
Number of Days in coupon period	=	365days in a year/2 coupons per year
	=	182.5days

The accrued interest is therefore:

$$A_{i,t}=\frac{105}{182.5} \times \frac{2.75\%}{2} =0.79110$$

Example 3 – 30/360

Under the 30/360 day count convention, it is assumed that there are 30 days in a month, and 360 days in a whole year. In order to calculate the period between two dates; first the number of months between the two dates is found and multiplied by 30; this is added to the difference between the day component of the dates. Using the same details as the bond in example 1, the calculation is as follows:

Settlement date – Previous Coupon Date	=	04 August 2014 – 21 April 2014
	=	(August – April) x 30 days + (4-21) days
	=	(4 * 30) days – 17 days
	=	103 days
Number of Days in coupon period	=	360 in a year/2 coupons per year
	=	180 days

The accrued interest is therefore:

$$A_{i,t}=\frac{103}{180}\times\frac{2.75\%}{2}=0.78681$$

Example 4 – ACT/365, following business day

Again using example 1, but with a settlement date of 07 Mar 2024. This means that the previous coupon date is 21 Oct 2023. As the previous coupon date falls on a Saturday, using the following business day convention, this date is moved to the next business day: 23 October 2023.

Settlement date – Previous Coupon Date	=	07 Mar 2024 – 21 Oct 2023
	=	136 days
Number of Days in coupon period	=	365days in a year/2 coupons per year
	=	182.5

The accrued interest is therefore:

$$A_{i,t}=\frac{136}{182.5}\times\frac{2.75\%}{2}=1.02466$$

Appendix 3: Further information

A Glossary of Terms used in FTSE Russell's Ground Rule documents can be found using the following link:

Glossary.pdf

Further information on FTSE Fixed income indices is available from FTSE Russell. For contact details please visit the FTSE Russell website or contact FTSE Russell client services at info@ftserussell.com.

Website: www.ftserussell.com