Factor exposure indexes

Index construction methodology

Introduction

The premise of a factor approach to indexes is to construct a stock index that has an intentional and greater exposure to a factor of interest than a given benchmark. By factor, we mean a stock level characteristic such as volatility or value represented by for example, Book to Price. When the benchmark is the Market Portfolio and a positive excess return (or factor premium) exists over the long term, this is often termed a "factor anomaly", contradicting as it does the "Efficient Market Hypothesis". Anomalies such as the "value effect" ([1], [2]), "size effect" ([1], [2]) and "momentum effect" ([3], [4]) are well documented. FTSE's aim is not to justify the existence of such anomalies, but to set out a systematic methodology for the construction of transparent indexes that exhibit an intentional and controlled exposure to the factor(s) of choice.

Interest in a factor approach has been rekindled. This stems from a belief that additional within asset class risk premia exist, and may be captured in a systematic low cost manner [8, 9]. Factor tilts represent a significant component of the value added by traditional active managers and factor approaches to diversification may be more effective in achieving diversification objectives [7] .

Alternatively weighted indexes frequently exhibit factor tilts. However, typically the objective of such indexes is not factor related. Diversification is the key to avoiding concentration and is the premise on which many alternatively weighted indexes, from Equal Risk Contribution to Maximum Diversification indexes are constructed. Minimum Variance indexes explicitly target reductions in index volatility. Whilst the objective of individual alternatively weighted indexes differs, a common thread is the presence of a specific, non-factor objective.

The historical performance of alternatively weighted indexes has generated huge debate. The consensus is that observed historical out-performance is driven largely by value and small capitalisation factor tilts along with rebalancing gains [5, 6]. This has resulted in a re-evaluation of the merits of factor based approaches to investing and has led some to conclude that alternatively weighted indexes are the appropriate mechanism for accessing factor risk premia.

This is overly simplistic; the majority of alternatively weighted indexes have very specific non-factor objectives; factor outcomes are incidental rather than the result of conscious design choices. The FTSE Minimum Variance indexes typically exhibit small capitalisation and low volatility tilts. The degree to which such tilts are systematic is debatable, but what is undeniable is that the index does not directly target either of these outcomes and the additional complexity of a minimum variance approach is unnecessary for the capture of any low volatility factor premium.

Style and factor indexes have been available for many years, but the role of these original products was largely to fulfill a benchmarking or performance and risk attribution function as opposed to representing replicable indexes, designed to capture the performance of specific factor objectives. FTSE believes that second generation factor indexes should employ a common construction methodology, that directly targets a specific factor objective, with due consideration for diversification, capacity and ease of replication.

Consequently, we emphasise that factor indexes and alternatively weighted indexes are not necessarily interchangeable. A factor index sets out to capture factor exposures in a controlled and considered way. An alternatively weighted index sets out to achieve some other goal and may incidentally achieve factor exposures. For example, a low volatility factor index with exposure to low volatility stocks is not the same as a minimum volatility index. This is clearly seen by noting that the pattern of cross-correlations may allow potentially volatile stocks to represent large weightings in a minimum volatility index, whereas such stocks must necessarily represent a relatively small proportion of any low volatility factor index. The factor index provides exposure to low volatility stocks, whereas the minimum volatility index minimises total index volatility which may (or may not) give incidental exposure to low volatility stocks.

Maximising exposure to a particular factor is not the only criterion of interest; factor indexes must also be "investable". That is, the degree of capacity, liquidity and concentration are also considerations. For example, if the sole objective is "maximum factor exposure" we would simply devise an index consisting of a single stock with the maximum factor value. Such a maximally undiversified index would be anathema to investors. Indeed the design of factor indexes is further complicated by the fact that factor objectives and investable characteristics are often inversely related. In what follows we demonstrate how to attain a balance across both of these features.

Finally, we address the question of how to create multiple-factor indexes; how does one ensure simultaneous exposure to more than one factor? When there is a reasonable positive correlation between factors, simple addition of individual factor index weights may be sufficient. However, when factors are anti-correlated, such an approach breaks down. To address this problem we introduce the notion of multiple-tilting.

The structure of this document is as follows. In Section 1 we discuss factor design. In Section 2 we set out a methodology for the construction of a single factor index by applying a "tilt" to the weights of some other (underlying) index. In Section 3 we discuss alternative approaches to constructing multiple-factor indexes. In Section 4 we consider the role and application of constraints. Section 5 investigates how the material set out in the previous four Sections may be applied to a concrete example. Finally we summarise our results and conclusions in Section 6.

1. Factor Design and Construction

1.1 Factor Definition

An important determinant of the behaviour of any factor index is the definition of the factor itself. For example should a value factor be based on a single valuation ratio or be comprised of a composite of different ones? Should such a composite comprise "equally-weighted" factors or should it be more heavily weighted to "more important" ones?

The answers to these questions are generally factor specific and preferably based on theoretical work supported by empirical results. The bulk of this document is concerned with what happens after a factor has been created; that is how it is turned into an index. We discuss the detailed construction of Value, Residual Momentum and Quality factors used in the FTSE Global Factor Index Series in a set of separate papers. However, there are some general techniques used during the construction of factors that are important to highlight.

1.2 Factor Neutralisation

Sometimes factor A will be highly correlated with another factor B. One may be interested in defining a "pure A factor" that is not confounded by a signal from factor B. There are various techniques that can be used to remove that part of factor A, which is strongly correlated to factor B.

As an example, consider the case where factor B represents the industrial grouping of a stock. More specifically, assume that factor A is correlated with particular industry groupings and it is desirable to remove this dependence such that any index does not reflect this industry bias. A simple approach is to redefine the stock level factor within a particular industry by measuring it relative to the mean value of the factor within that industry. This ensures all industries are equivalent from a factor perspective. Since the factor is now a relative measure, industry biases in any index based on the re-defined factor will be limited. We discuss a possible application of this, which aids the implementation of Industry and Country constraints, in Section 5.

More sophisticated approaches, which identify independent country and industry factor effects and assess the significance of these effects, are possible.

An alternative approach to limiting unintended factor positions is to apply constraints during the index construction process. We discuss possible approaches to implementing constraints in Section 4.

1.3 Composite Factors

There are several approaches to simultaneously achieving tilts towards multiple factors. The distinction between forming an index based on a composite of indexes and a composite factor is discussed in Section 3.

2. Single Factor Index Methodology

2.1 Calculation of Factor Z-Scores

Consider a universe of stocks U and let f_i be some factor that takes real values for stocks within some subset F of U. Then we define the Z-Score of the factor for stocks in the usual way:

$$z_i = \frac{f_i - \mu}{\sigma}$$

where μ and σ are the cross-sectional mean and standard deviation of the factor. Stocks with Z-Scores less than minus three are set to minus three and those with Z-Scores greater than three are set to three. The Z-Scores are then recalculated. This process is repeated iteratively until all remaining stocks have absolute Z-Scores less than or equal to three.

2.2 Mapping Z-Scores to Scores

Z-Scores are now assigned a score, $S_i \in (0, \infty)$ to determine the weights in the factor index. The functional form we use is based on the cumulative normal:

$$s_i = CN(Z_i) = \int_{-\infty}^{z_i} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx$$

If the whole universe is to be weighted, stocks in $U - F$ (i.e. without real factor values) are assigned the score $CN(-3)$ or alternatively assigned a neutral value of $CN(0)$.

The cumulative normal mapping has several advantages over other commonly used mappings, such as:

$$S_i = M(Z_i) = \begin{cases} \dfrac{1+Z_i}{2} & \text{if } Z_i \geq 0 \\ \dfrac{1}{2(1-Z_i)} & \text{if } Z_i < 0 \end{cases}$$

An additional mapping that is of interest is:

$$S_i = V(Z_i) = \begin{cases} f_i & \text{if } f_i > 0 \\ f_{min} & \text{if } f_i \leq 0 \end{cases}$$

for some chosen minimum value f_{min}.

This mapping can be used to derive a set of "Value Weighted" or "Fundamentally Weighted" indexes, an example of which is discussed in Section 5.

Chart 1 compares mapping schemes based on the cumulative normal $CN(Z)$ and $M(Z)$. The main difference between these two mapping schemes lies in the tails of the Z-Score data. $M(Z)$ tends to overweight stocks with extreme Z-Scores compared to $CN(Z)$. This is undesirable, since extreme Z-Scores are likely to be increasingly unreliable and result in reversals of index weights between index reviews generating unnecessary turnover.

Chart 1. Z-Score Mapping Schemes

Mapped Scores (y-axis), _Z-Score_ (x-axis)

— M(Z)   — CN(Z)
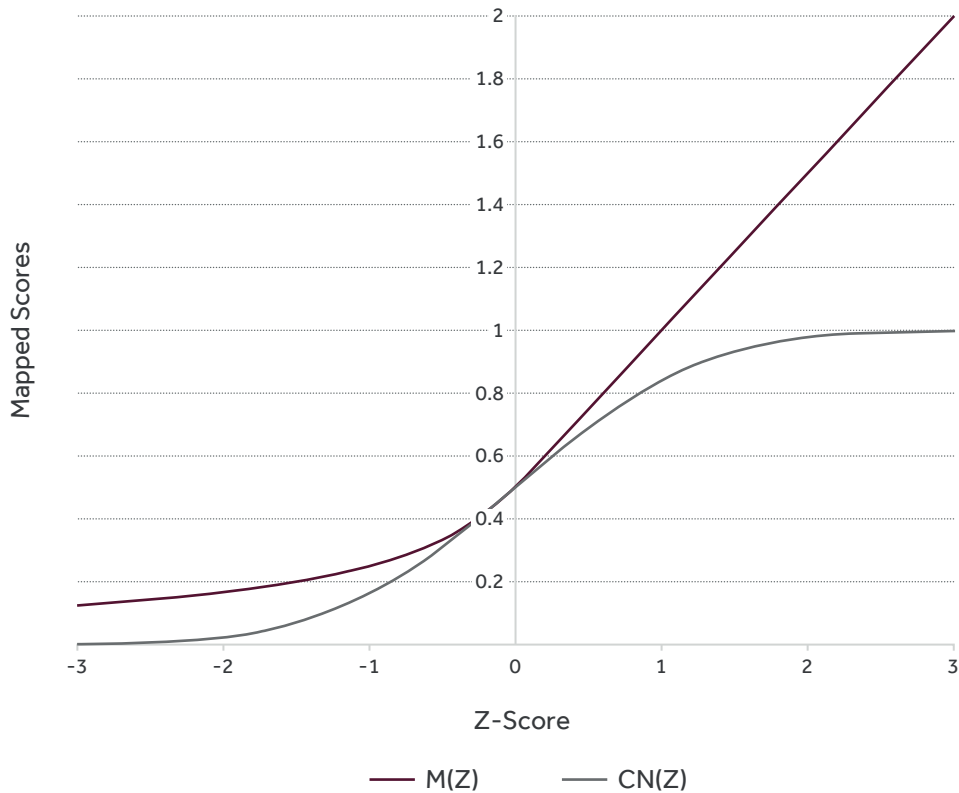
2.3 Cumulative Normal Compared to Rank Based Scores

Where a factor is independently and identically normally distributed, the application of the cumulative normal mapping function will yield identical results to the application of a rank based scoring approach in the limit for large sample sizes.

However, where this distributional assumption does not hold and/or for smaller sample sizes, there is a subtle difference between the two approaches; the cumulative normal considers the magnitude of the Z-Score, not just the ranking. This is seen more clearly when one considers that adjacent stocks will have trivially different rankings, whereas the cumulative normal interval between adjacent stocks may be significant.

2.4 Translating Scores to Index Weights

For stocks in the universe U with underlying index weights of W_i, the factor index weights are:

$$\hat{W}_i = \frac{S_i * W_i}{\sum_{j \in U} S_j * W_j}$$

The underlying index weights may be of any type; for example they may be Market Capitalisation, Equal or Risk Weights. The resulting factor index can therefore be considered as a "factor overlay" or "factor tilt" on an underlying index. At this stage a minimum weight may be imposed or weights below a minimum threshold set to zero.

To illustrate, consider an equally-weighted underlying index consisting of 1000 stocks and a hypothetical factor whose values have been taken from the normal distribution with mean zero and standard deviation one. Chart 2 shows the resulting distribution of index weights after the application of the *CN(Z)* and *M(Z)* mapping schemes. Stocks are ordered left to right from the smallest to largest factor score. Note that *M(Z)* overweights the tails as expected and shows a potentially problematic concentration in weight for stocks with very large Z-Scores. On the other hand *CN(Z)* yields a straight line relationship and therefore avoids such concentrated outcomes.

Chart 2. Translating Scores to Weights
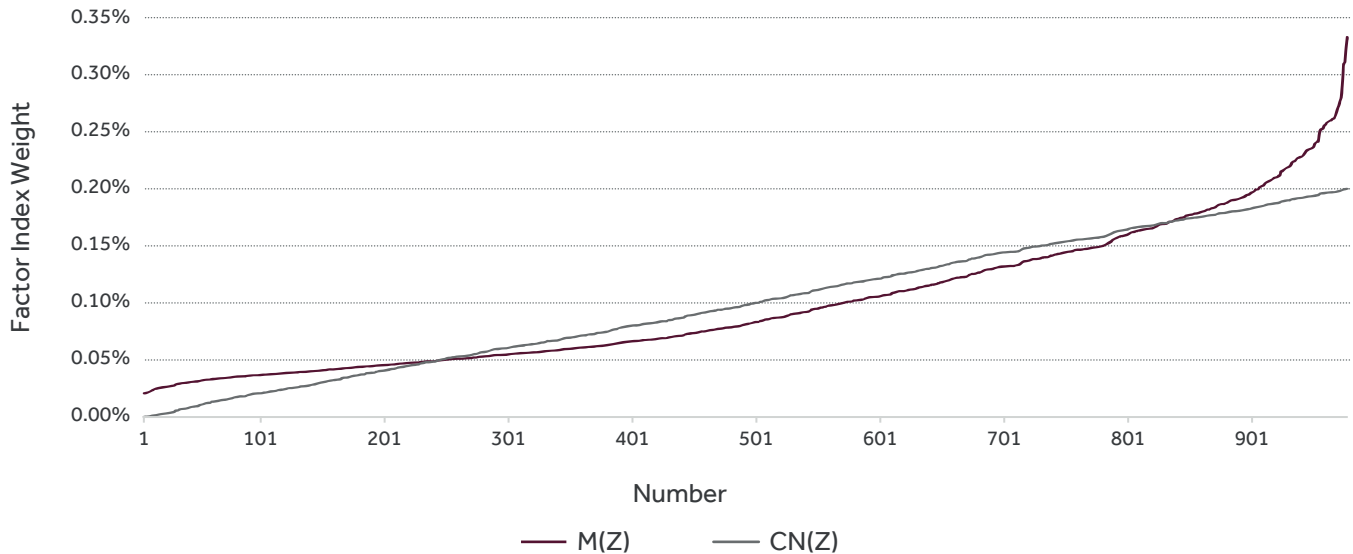
The transfer coefficient, representing the efficiency with which the factor signal is reflected in the resulting factor index when *M(Z)* is used is high at 95%. However, it is higher still when *CN(Z)* is used, at 98%. In the limit of large sample size the theoretical transfer coefficient values can be shown to be 95.34% and 97.72% ($\sqrt{3/\pi}$) respectively.

2.5 Direction of Factor Tilt

A factor index may be tilted in either direction, for example, a long-only low momentum index can be created as opposed to a long-only high momentum index. To create an index that tilts away from a given factor, the sign of a stock's Z-Score is simply reversed and the score calculated:

$$S_i = CN(-Z_i)$$

This enables the implementation of any view on the prospects of a factor in a long-only context. That is, one can be short a factor without needing to be short the index.

The cumulative normal approach has the additional property that a tilt away from a factor satisfies a symmetry relationship with a tilt towards the same factor. The condition that $CN(Z_i) + CN(-Z_i) = 1$ ensures that a linear sum of the stock weights of a factor index tilted towards a factor and one tilted away will yield the underlying index weights. The *M(Z)* scoring function does not share this property.

2.6 Controlling the Strength of a Factor Tilt and Multi-tilting

There are (at least) two ways that the strength of the factor tilt can be altered. Firstly a cumulative normal mapping that has a different standard deviation parameter may be used. The smaller the standard deviation, the stronger the resulting factor tilt. In the limit of zero standard deviation, the cumulative normal function becomes a step function, with zero for negative Z-Scores and one for positive Z-Scores. This results in indexes that consist only of those stocks in the top half of the universe by factor rank, where weights are in identical proportion to the underlying index.

The second method is to perform a "tilt on a tilt". First we tilt our underlying index toward the factor in the usual way. The resulting weights are now considered as a "new underlying" and are tilted towards the factor again. In this way we create a stronger tilt towards the factor and may continue the process to yield even stronger tilts. Mathematically this can be seen as "exponentiation of the tilt operator".

We are not forced to use the same factor at each step in the process and therefore can create indexes that are tilted towards more than one factor. It is trivial to show that the order of tilting makes no difference to index outcomes, since a stock's score in a tilt-tilt index is equal to the product of the separate factor scores. Furthermore, this can be shown explicitly for a multi-tilted index based on the cumulative normal by noting that the score for a stock is given by:

$$S = CN(Z_1, Z_2) = \int_{-\infty}^{Z_1} \int_{-\infty}^{Z_2} \frac{e^{-(x^2+y^2)/2}}{2\pi} dx\, dy$$

where Z_1 and Z_2 are the stock's Z-Scores for factor one and factor two respectively.

2.7 Long/Short Factor Indexes

From an implementation perspective, long-only indexes are preferable to long/short factor indexes. However, long/short factor indexes will provide a stronger tilt towards a factor of interest. A long/short approach can be incorporated within the framework described by combining a long position in a positively tilted index with a short position in the corresponding negatively tilted index. The symmetry property of the Cumulative Normal discussed in Section 2.5 ensures that a short negative factor tilt could at least theoretically be created, by being short the underlying index (future) and long the positive factor index. Chart 3 shows the distribution of long/short weights when this approach is applied to an equally-weighted underlying index with hypothetical factor scores drawn from the normal distribution with mean zero and standard deviation one.

Chart 3. Long/Short Factor Index Weights
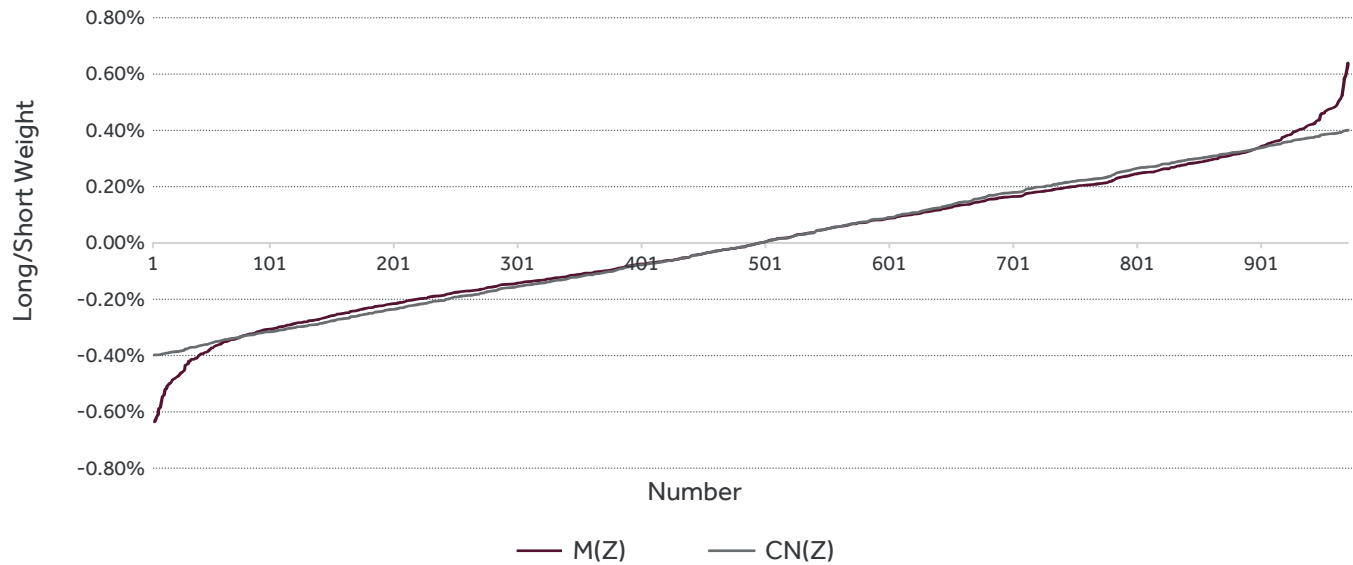
We have fixed the size of the long position to be equal to one. Note that, unlike *M(Z)*, the cumulative normal avoids concentration in weight for extreme positive and negative Z-Scores.

2.8 Broad and Narrow Indexes

The size and composition of the underlying universe has a profound effect on the characteristics of a tilted index. For large numbers of stocks with suitably "well-behaved" factors we envisage no difficulty in applying Cumulative Normal scoring. For small numbers of stocks with unusual distribution of factor values the application of the Cumulative Normal may lead to excessively concentrated weightings. A possible solution to this is to apply a ranking tilt (see Section 2.3) for small sample sizes. This is consistent with the Cumulative Normal approach, since in some overlapping regions of sample size the two will yield indexes of similar composition.

Alternatively one could turn to optimised strategies which control for factor exposure, diversification and concentration. This has the advantage that one has precise control over index outcomes but at the expense of less transparency. These issues and approaches to constructing intentionally narrow indexes displaying very high levels of factor exposure will be examined at a later date. The intention is to provide a mechanism for creating indexes with very high exposure that are relatively easy to replicate. We anticipate that such indexes may be suitable for factor ETFs and provide a mechanism for fund managers to efficiently add or remove a tactical factor overlay to an existing portfolio.

In contrast, broad high capacity factor indexes would appear to be more suitable for long-only asset managers and owners with a strategic perspective on the existence of factor risk premia. The approach to factor index construction detailed to date, results in the reweighting of all constituents in an underlying index, irrespective of whether they contribute towards achieving a given factor objective.

A simple method that can be used to produce factor indexes with fewer stocks is to remove stocks that contribute only trivially to the overall factor objective. One can achieve this by removing stocks in order of weight, factor score or weight*factor

score, (smallest first) while confirming that the effective number of stocks (diversification) and investable characteristics (capacity and liquidity) of the index does not fall below some pre-specified values (see Appendix B for definitions of these). We illustrate the application of such an approach in section 5.9.

3. Combining Factors

3.1 Composite Indexes

Where a simultaneous tilt towards two or more factors is required, there are several different ways to determine an appropriate set of index weights. The composite index approach simply takes the weights derived from M separate factor indexes and combines them:

$$\hat{W}_i = \sum_{j=1}^{M} \alpha_j * W_i^j$$

Where W_i^j is the weight of the ith stock in the jth factor index and : α_j are real positive numbers satisfying $\sum_j^N \alpha_j = 1$. The α_j determine the relative strength of the tilts to our various factors. For example if an "equal tilt" towards all factors is required we set α_j = 1/M.

3.2 Composite Factors

A second approach is to create a composite factor F_i based on Z-Scores calculated for each single factor as described in Section 2:

$$F_i = \sum_{j=1}^{M} \alpha_j * Z_i^j$$

where Z_i^j is the Z-Score of the ith stock in the jth factor index. The problem then reduces to a single (composite) factor and the methods of Section 2 can be employed to determine the factor index weights.

3.3 Discussion

Both approaches are readily implemented within the methodology outlined. Intuitively, the composite factor approach is preferable as it is conceivable that combining factor indexes in the manner of Section 3.1, where the underlying factors are negatively correlated may result in an index that fails to achieve a tilt in any of the desired directions.

Conversely, the composite factor approach offers the flexibility to achieve the multiple factor tilt objective through transformations of the input factor Z-Scores and selection of the appropriate factor weightings. The use of a composite factor allows us to redesign factors in the manner discussed in Section 1 and incorporate them into a general factor index methodology.

Finally multiple factor objectives may also be constructed using the multiple tilt methodology detailed in Section 2.6. We illustrate the application of multiple tilts in Section 5.4.

4. Constraints

4.1 Industry and Country Constraints

A factor may be strongly correlated with industries and or countries. Consequently, the construction process described may give rise to unintentional over/under weight positions in particular industries or countries. For example, a dividend yield factor index will preferentially tilt towards high yield industries, e.g. utilities and away from low yield countries, e.g. Japan. This may be desirable, but it is not clear that the resulting index reflects the performance of a dividend yield tilt as opposed to country or industry effects.

One solution is to redefine the factor in order to limit such biases. This involves the industry/country neutralisation methods described in Section 1. However even after the application of such a factor redefinition, the resulting index may still contain (smaller) biases. Consequently a general method is required to correct for this.

The objective is to constrain the factor index, such that industry and country weightings do not deviate too much from those in the underlying index. One approach to achieving this objective is to impose a requirement that all industry and country weights in the factor index are bounded by:

$$Max[\{(100-p)*W-q\},0] \ and \ Min[\{(100+p)*W-q\}, \ 100]$$

for each underlying industry/country weight W and some chosen percentage values p and q. For example p = 10 requires the factor index to have weights between +/- 10% of the underlying index weights. Furthermore q = 5 provides additional freedom to assume meaningful active weights of +/-5% in small industries/countries, subject to the requirement that the index has no short positions.

Initially, we construct a factor index using the methods outlined in earlier sections and assess whether the index satisfies all constraints. If not, a simple transparent method of imposing such constraints is required. Here we propose two such methods.

4.1.1 Iterative Application of Constraints

One approach is to set the weight of breaching industries and countries to the nearer of their upper or lower bounds. Weight is then re-assigned proportionately to industries/countries that were not in breach of their upper or lower bounds. Where such a reallocation causes breaches in previously "good" industries and countries, then the original constraints are relaxed until no such breaches occur. For example the constraint (p,q) = (10,5) may be relaxed to (p, q) = (10.1,5.1), (10.2.5.2), etc, until the redistribution of weight causes no breaches to occur in any of the good industries and countries.

A final iteration is performed to ensure consistency between the newly constrained country and industry positions.

4.1.2 A Composite Index to Satisfy Constraints

A second approach relies on the creation of a composite index. The composite index consists of a mix of the unconstrained factor index and the underlying index and progressively tilts away from the underlying index using the methodology of Section 3.1, whilst respecting all constraints.

Consider two sets of weights; those of a factor index and those of the underlying index. Starting with $\alpha_I = 0$ for the factor index and $\alpha_U = (1 = \alpha_I) = 1$ for the underlying index, we increase α_I until any one of the constraints is breached. This determines the maximum α_I (or factor tilt) that is consistent with all the constraints. The looser the constraints, the greater the potential tilt.

4.1.3 Discussion

The factor tilt method described in Section 4.1.2 has the advantage that it is both intuitive and transparent. However, we believe that greater factor tilts for a given set of constraints are possible using the iterative method in Section 4.1.1.

Our reasoning is as follows: in the iterative case we look for perturbations of the unconstrained factor index that satisfy the constraints, i.e. we move from the unconstrained factor index to a close suitable index.

In contrast, the starting point of the factor tilt method of Section 4.1.2 may be a significant distance from the unconstrained factor index, (i.e. the underlying index) and moves along a pre-defined path towards an index that nearly breaches one of the constraints. There is always the possibility that another path would lead to a solution that is closer to the unconstrained factor index.

We examine empirically these approaches to applying constraints in Section 5.6.

5. Factor Index Example

In this Section we apply the methods discussed in previous Sections to create a simple value factor index premised on Earnings Yield (E/P). We construct indexes based on each of the three scoring methods set out in Section 2 using both a capitalisation weighted and an equally-weighted underlying index. The underlying universe consists of constituents of the FTSE Developed Index. Unless otherwise stated, for illustration, indexes are rebalanced on a monthly basis from May 2000 to October 2013. All performance figures are in US dollars.

5.1 Performance Summary

Table 1 shows the results of applying each of the alternative mapping schemes to a market capitalisation weighted underlying index. Note that where scores are based on raw (positive) values of E/P, the resulting index is equivalent to an index where individual index weights are proportional to absolute earnings. That is, the index is "Fundamentally" or "Value-Weighted".

Table 1. Market Capitalisation Weighted Underlying Index

	FTSE developed	Cumulative normal – CN(Z)	Alternative approach – M(Z)	Value weighted – V(Z)
Geometric Mean (%)	1.66	3.26	3.11	2.83
Volatility (%)	17.14	17.47	17.85	18.08
Volatility Reduction (%)		-1.90	-4.14	-5.48
Sharpe Ratio	0.10	0.19	0.17	0.16
DD (%)	-58.95	-59.40	-60.94	-60.09
Two Way Turnover (%)		106.20	120.33	149.69
Excess (%)		1.58	1.43	1.16
Tracking Error (%)		2.09	2.46	4.61
Information Ratio		0.76	0.58	0.25
Alpha (%)		1.59	1.44	1.25
Alpha T-Stat		2.84	2.23	1.01
Beta		1.01	1.03	1.02
Capacity (WCR)	1.00	1.30	1.43	1.55

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

All performance metrics are similar, though slightly better for *CN(Z)* and *M(Z)* than for *V(Z)*. However, as anticipated, turnover under the cumulative normal approach is smaller than when *M(Z)* is used and both approaches result in substantially lower turnover than the "Value-Weighted" approach. The cumulative normal achieves this reduction in turnover by ensuring that stocks with extreme Z-Scores are not significantly (and unnecessarily) re-weighted each month as a result of noise in the factor. We also note that the Weighted Capacity Ratio (WCR See Appendix B) is superior for *CN(Z)* than for *M(Z)* and *V(Z)*.

Table 2 presents the comparable results using equally-weighted underlying index. Note that *V(Z)* now delivers something different from a Value-Weighted Index.

The turnover of all mappings is roughly one and a half times that seen when each factor mapping is applied to a capitalisation weighted underlying index. The performance figures for the cumulative normal and *M(Z)* approaches are broadly similar, but now diverge from those of *V(Z)*. The cumulative normal approach continues to deliver noticeably lower turnover. As expected, capacity deteriorates for all scoring schemes, but is noticeably superior under the cumulative normal approach.
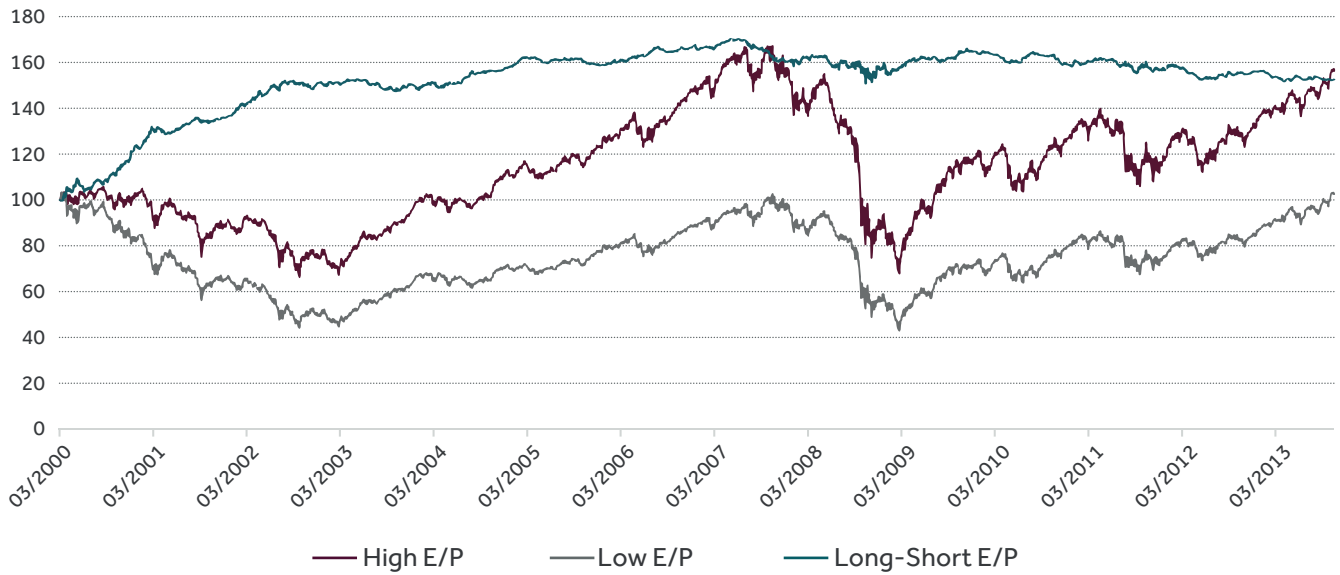
Table 2. Equally-Weighted Underlying Index

	FTSE developed equally-weighted	Cumulative normal – CN(Z)	Alternative approach – M(Z)	V(Z)
Geometric Mean (%)	6.08	7.98	8.14	7.04
Volatility (%)	15.90	16.15	16.63	17.30
Volatility Reduction (%)		-1.56	-4.55	-8.77
Sharpe Ratio	0.38	0.49	0.49	0.41
DD (%)	-58.56	-59.95	-61.39	-60.23
Two Way Turnover (%)	96.17	171.03	187.46	245.27
Excess (%)		1.79	1.93	0.90
Tracking Error (%)		2.18	2.52	5.95
Information Ratio		0.82	0.77	0.15
Alpha (%)		1.77	1.79	0.97
Alpha T-Stat		3.02	2.71	0.61
Beta		1.01	1.03	1.02
Capacity (WCR)		10.99	20.94	149.32

Source FTSE: FTSE Developed; equally-weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

5.2 Tilting Towards High And Low Earnings Yield

To create a long index tilted towards low earning yield stocks (E/P); we reverse the sign of the Z-Score used to derive factor scores and derive index weights by applying the *CN(Z)* mapping. Chart 4 illustrates the performance of unconstrained versions of such indexes based on a capitalisation weighted underlying index and Table 3 details the summary performance metrics.

Chart 4. High and Low E/P Tilts and a Long/Short E/P Index

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns;
March 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may
reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

The long/short index is obtained by taking a long position in the positive earnings
yield tilt index and short position in the negatively tilted earnings yield index.

Table 3. High and Low Earnings Yield: Factor Performance

	FTSE developed	High E/P positive tilt	Low E/P negative tilt	Long – short
Geometric Mean (%)	1.66	3.26	0.70	2.54
Volatility (%)	17.14	17.47	16.95	4.04
Volatility Reduction (%)		-1.90	1.08	76.42
Sharpe Ratio	0.10	0.19	0.04	0.63
DD (%)	-58.95	-59.40	-58.09	-11.55
Two Way Turnover (%)		106.20	115.12	–
Excess (%)		1.58	-0.94	0.87
Tracking Error (%)		2.09	2.04	17.11
Information Ratio		0.76	-0.46	0.05
Alpha (%)		1.59	-0.92	2.50
Alpha T-Stat		2.84	-1.71	2.32
Beta		1.01	0.98	0.03

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000
to October 2013. Past performance is no guarantee of future results. Returns shown may reflect
hypothetical historical performance. Please see page 36 for important legal disclosures.

5.3 Varying the Strength of the Factor Tilt

Table 4 illustrates the effect of increasing the strength of the earnings yield tilt away from a market capitalisation weighted underlying index, using both the methods described in Section 2.6. The column named "Tilt with StDev = 1.0" represents our base case earnings yield index. "Tilt with StDev = 0.5" (or 0.1) represents the index obtained using a cumulative normal with standard deviation parameter equal to 0.5 (0.1). Finally "Tilt-Tilt" is the index resulting from two consecutive tilts towards earnings yield.

Table 4. Varying Tilt Strength: Factor Performance

	Tilt with StDev = 1.0	Tilt with StDev = 0.5	Tilt with StDev = 0.1	Tilt – Tilt
Geometric Mean (%)	3.26	4.19	5.09	4.25
Volatility (%)	17.47	17.68	17.97	18.15
Volatility Reduction (%)	-1.90	-3.15	-4.82	-5.88
Sharpe Ratio	0.19	0.24	0.28	0.23
DD (%)	-59.40	-59.65	-60.27	-61.15
Two Way Turnover (%)	106.20	140.46	184.28	164.94
Excess (%)	1.58	2.49	3.37	2.55
Tracking Error (%)	2.09	2.95	3.74	3.58
Information Ratio	0.76	0.85	0.90	0.71
Alpha (%)	1.59	2.50	3.38	2.58
Alpha T-Stat	2.84	3.18	3.39	2.73
Beta	1.01	1.02	1.03	1.04
Capacity (WCR)	1.30	1.56	2.04	1.81
Value Loading	0.59	0.78	0.95	0.96

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

Decreasing the standard deviation of the cumulative normal mapping results in increasingly strongly tilted indexes as evidenced by the increased factor premium and value loading (see Appendix A). The Tilt-Tilt index results in similar performance and higher turnover figures to those obtained from a single tilt with standard deviation equal to 0.5, but exhibits a greater value exposure. The Tilt-Tilt index results in comparable loading on the value factor, at lower levels of turnover and improved levels of index capacity compared to a "Tilt with StDev = 0.1".

5.4 Composite Factor Indexes versus Composite Indexes

In this subsection we combine Earnings Yield (E/P) and Book to Price (B/P) factors to form a single "Value Index". From Section 3 we can do this in at least two ways; by constructing a (equally-weighted) composite factor or by simply combining the weights of the separate indexes.

Table 5. Combining Factors: Value and Value

	FTSE developed	Earnings yield	Book to price	Composite factor	Weight combination/ composite index
Geometric Mean (%)	5.35	6.02	5.58	5.91	5.81
Volatility (%)	17.27	17.76	18.52	18.10	18.09
Volatility Reduction (%)		-2.86	-7.22	-4.80	-4.75
Sharpe Ratio	0.31	0.34	0.30	0.33	0.32
DD (%)	-58.95	-59.40	-63.46	-61.15	-61.43
Two Way Turnover (%)		104.98	88.40	103.93	92.30
Excess (%)		0.63	0.22	0.53	0.44
Tracking Error (%)		1.89	3.12	2.24	2.16
Information Ratio		0.33	0.07	0.24	0.20
Alpha (%)		0.56	0.05	0.40	0.31
Alpha T-Stat		1.08	0.06	0.67	0.54
Beta		1.02	1.06	1.04	1.04

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; September 2001 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

The two approaches result in very similar outcomes and since Earnings Yield and Book to Price are strongly positively correlated, the performance of both composites is very similar to their component parts.

We now create a composite index comprising negatively correlated factors. Specifically, we combine Earnings Yield and a momentum measure (12-Month Price Return). The results are shown in Table 6.

The Composite Factor and Composite Index outcomes are similar, exhibiting smaller tracking errors than either of the component indexes. The negatively correlated Earnings Yield and Momentum factors have therefore "cancelled one another out" resulting in an index that is closer to the underlying. Indeed the loading to value and momentum (see Appendix A) are roughly half those of the individual factor indexes.

The final column in Table 6 shows the result of tilting the underlying index towards Earnings Yield and then tilting the resulting index towards Momentum (or equivalently vice versa). The tracking error of this index is approximately the same size as that of the Momentum index. This suggests that the index does not result in off-setting factor exposures and the index genuinely differs from the underlying index. That this is a genuine Value-Momentum index is confirmed by the fact that the loadings of this index on value and momentum are roughly double that of the two composite alternatives and comparable to that of the single factor indexes.

This much improved loading profile however comes at the price of higher turnover and a reduction in capacity.

Table 6. Combining Factors: Value and Momentum

	Earnings yield	12-Month price return	Composite factor	Weight combination/ composite index	Tilt – Tilt
Geometric Mean (%)	6.02	5.51	5.70	5.79	6.08
Volatility (%)	17.76	16.53	17.01	17.05	16.85
Volatility Reduction (%)	-2.86	4.27	1.49	1.25	2.40
Sharpe Ratio	0.34	0.33	0.34	0.34	0.36
DD (%)	-59.40	-56.29	-58.02	-57.83	-56.37
Two Way Turnover (%)	104.98	206.08	142.09	124.00	248.33
Excess (%)	0.63	0.16	0.33	0.42	0.69
Tracking Error (%)	1.89	2.68	1.54	1.34	2.60
Information Ratio	0.33	0.06	0.22	0.31	0.27
Alpha (%)	0.56	0.39	0.42	0.48	0.86
Alpha T-Stat	1.08	0.55	0.98	1.30	1.20
Beta	1.02	0.95	0.98	0.98	0.97
Capacity (WCR)	1.29	1.26	1.17	1.12	1.53
Value Loading	0.59	-0.01	0.30	0.29	0.51
Momentum Loading	0.01	0.47	0.28	0.24	0.48

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; September 2001 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

5.5 Industry and Country Weights

Charts 5 and 6 illustrate the industry and active country weightings from the application of the cumulative normal approach to the Earnings Yield factor and the capitalisation weighted FTSE Developed index as of June 2013. Note no constraints or additional normalisation has been applied.

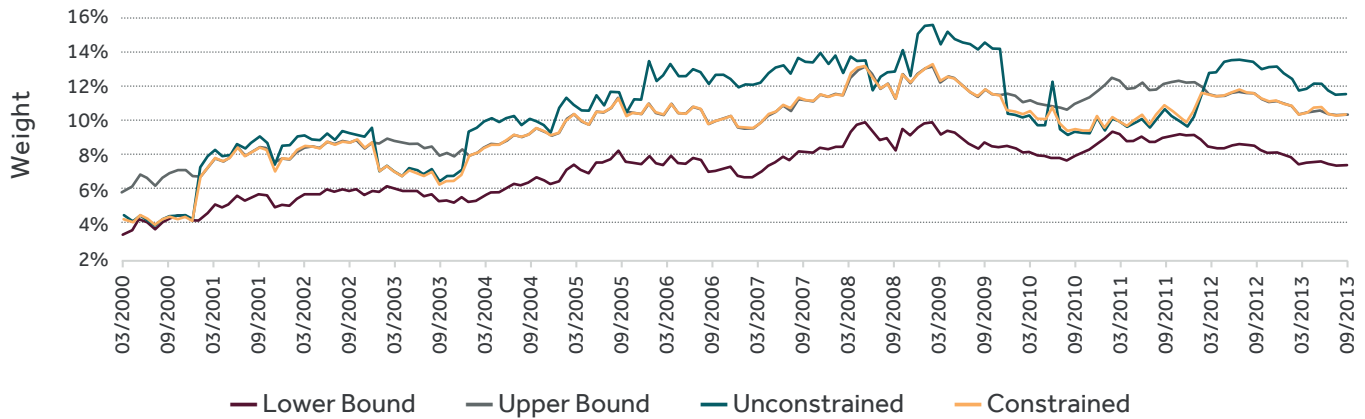**Chart 5. Industry Weightings for June 2013**
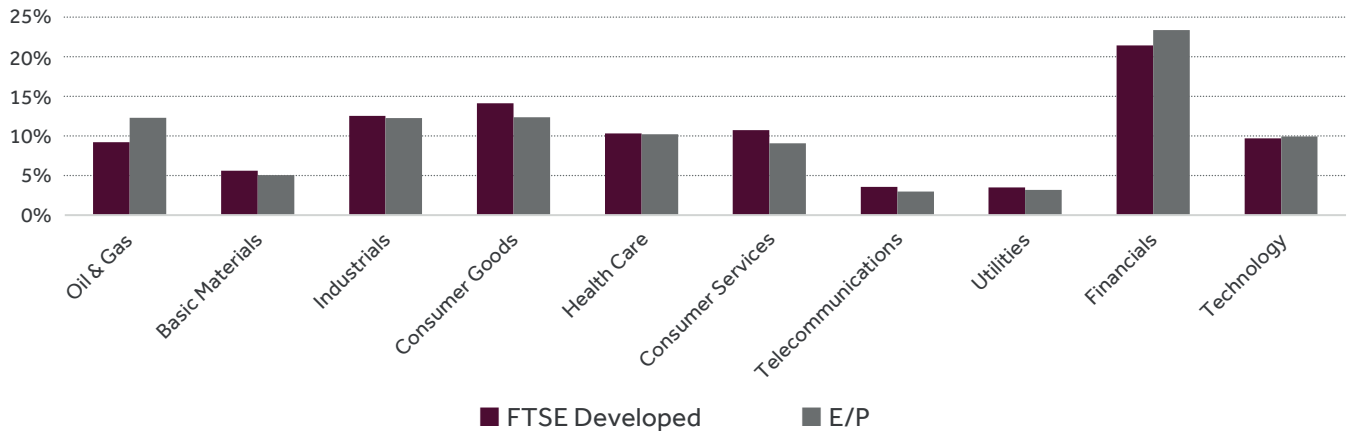
Source FTSE: Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

If we were to use the constraints model of Section 4.1 with $(p,q) = (5,1)$, then the June 2013 weights shown in Charts 5 and 6 breach the respective Oil & Gas industry and Japan country limits.

Chart 6. Active Country Weights for June 2013
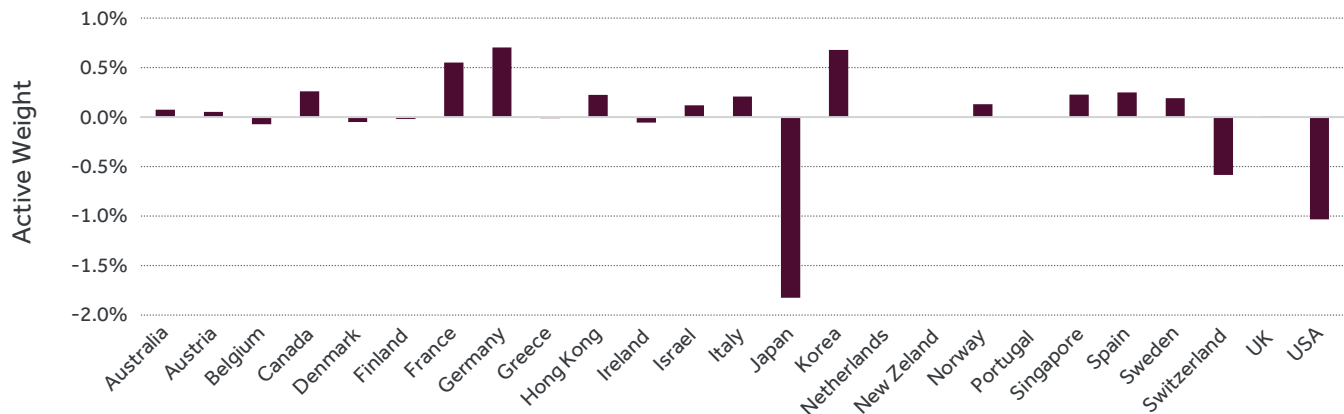
Source FTSE: Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 34 for important legal disclosures.

5.6 Application of Constraints

We now impose the $(p,q) = (5,1)$ country and industry constraints detailed in Section 4.1. Chart 7 illustrates how the constraint on Oil & Gas is breached by our unconstrained factor index through time and how the iterative technique set out in Section 4.1.1 remedies this.

We choose the iterative application of constraints in contrast to creating the tilted composite index detailed in section 4.1.2. In this case, this is a superior option, since the mean sum of absolute differences between the constrained and unconstrained weights through time are 9.2% and 27.9% respectively. The iterative application of the constraints results in outcomes that are substantially closer to the unconstrained factor index than the tilted composite index approach.

Chart 7. Constraint Breaches for Oil & Gas

Legend: — Lower Bound   — Upper Bound   — Unconstrained   — Constrained

Source FTSE: March 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

Table 7 illustrates the effect of increasingly stringent constraints on the performance characteristics of the Earnings Yield factor index between May 2000 and October 2013. Note, that as the constraints become more stringent, the tracking error between the factor and underlying index becomes smaller; additionally, both the value loading and the factor premium (as measured by the excess return) also decline. This is expected as the constraints force the factor index back towards the underlying (market capitalisation weighted in this case) index.

Table 7. Iterative Application of Constraints to a Factor Index

	FTSE developed	No constraint	(10, 5)	(5, 2)	(5, 1)	(2, 1)
Geometric Mean (%)	1.66	3.26	3.26	3.09	2.96	2.91
Volatility (%)	17.14	17.47	17.47	17.41	17.34	17.31
Volatility Reduction (%)		-1.90	-1.92	-1.58	-1.19	-1.02
Sharpe Ratio	0.10	0.19	0.19	0.18	0.17	0.17
DD (%)	-58.95	-59.40	-59.39	-59.09	-59.08	-58.95
Two Way Turnover (%)		106.20	106.18	107.21	108.88	110.38
Excess (%)		1.58	1.58	1.41	1.28	1.23
Tracking Error (%)		2.09	2.05	1.76	1.59	1.45
Information Ratio		0.76	0.77	0.81	0.81	0.85
Alpha (%)		1.59	1.58	1.42	1.28	1.23
Alpha T-Stat		2.84	2.89	3.02	3.03	3.17
Beta		1.01	1.01	1.01	1.01	1.01
Value Loading	0.00	0.59	0.59	0.52	0.46	0.43

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

Note that when constraints are lax, for example, (p, q) = (10, 5) , outcomes differ very little from the unconstrained outcomes. In this case the constraints act as "insurance" against taking very large under/overweight positions relative to an underlying index.

5.7 Additional Normalisation

We now demonstrate that the industry and country active weights may also be controlled by a more sophisticated design of the raw factor. Earnings Yield is now measured relative to industrial group membership in the manner described in Section 1.2.

Chart 8 illustrates that such an industry normalised factor results in fewer and smaller breaches of the *(p,q)* = (5,1) constraint for Oil & Gas (unconstrained line) than the non-normalised factor shown in Chart 7.

The constrained line in Chart 8 shows the Oil & Gas industry weights after the application of the *(p,q)* = (5,1) constraint to the industry normalised factor. The constrained and unconstrained lines are almost coincident indicating that the newly designed factor already adequately controls for industry exposure. Indeed the average absolute weight difference through time between the constrained and unconstrained index weights is only 2.2%.

Chart 8. Constraint Breaches for Oil & Gas: Additional Normalisation
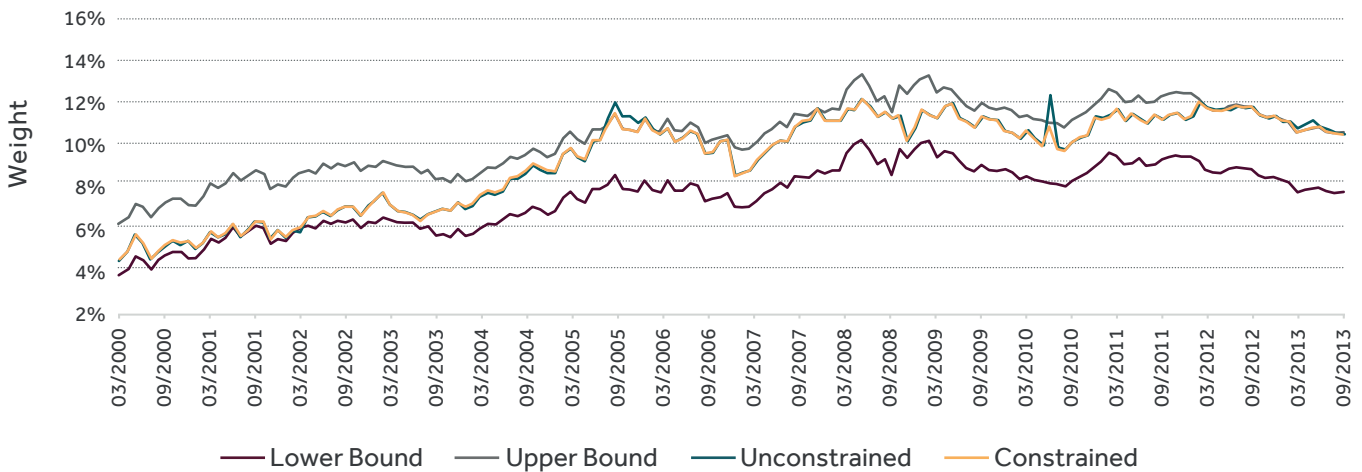
Source FTSE: March 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

Table 8 shows the performance of non-normalised and industry normalised Earnings Yield factor indexes both before and after the application of constraints. There is relatively little difference in the performance of the unconstrained and constrained industry normalised approaches and both exhibit lower tracking errors and inferior performance compared to the equivalent non-normalised industry approaches. Note also that, as one would expect, an industry normalised approach results in a smaller loading on value.

Table 8. Industry Normalised and Iterative Constraint Approaches: EY Factor Performance

	FTSE developed	Industry normalised approach		No Industry normalisation approach	
		Unconstrained	Constrained (5, 1)	Unconstrained	Constrained (5, 1)
Geometric Mean (%)	1.66	2.39	2.41	3.26	2.96
Volatility (%)	17.14	17.37	17.29	17.47	17.34
Volatility Reduction (%)		-1.37	-0.90	-1.90	-1.19
Sharpe Ratio	0.10	0.14	0.14	0.19	0.17
DD (%)	-58.95	-58.70	-58.64	-59.40	-59.08
Two Way Turnover (%)		99.33	99.93	106.20	108.88
Excess (%)		0.72	0.74	1.58	1.28
Tracking Error (%)		1.29	1.16	2.09	1.59
Information Ratio		0.56	0.64	0.76	0.81
Alpha (%)		0.72	0.74	1.59	1.28
Alpha T-Stat		2.11	2.40	2.84	3.03
Beta		1.01	1.01	1.01	1.01
Value Loading	0.00	0.29	0.28	0.59	0.46

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

5.8 Cumulative Normal versus Rank Scoring

As noted in Section 2, in the limit of a normally distributed factor and large sample size the Cumulative Normal and simple Rank scoring will yield identical outcomes. However, factors are unlikely to be normally distributed and sample sizes are limited. Indeed for the Earnings Yield factor using the FTSE Developed universe, (approximately 2000 stocks) one can show that the average absolute weight difference through time for indexes constructed using each approach is 4.8%. The respective performance statistics for indexes constructed from a market capitalisation weighted underlying index are shown in Table 9.

Table 9. Cumulative Normal versus Rank Scoring

	FTSE developed	Cumulative normal	Rank
Geometric Mean (%)	1.66	3.26	3.19
Volatility (%)	17.14	17.47	17.40
Volatility Reduction (%)		-1.90	-1.50
Sharpe Ratio	0.10	0.19	0.18
DD (%)	-58.95	-59.40	-59.24
Two Way Turnover (%)		106.20	99.19
Excess (%)		1.58	1.51
Tracking Error (%)		2.09	1.94
Information Ratio		0.76	0.78
Alpha (%)		1.59	1.51
Alpha T-Stat		2.84	2.92
Beta		1.01	1.01
Capacity (WCR)	1.00	1.30	1.26
Value Loading	0.00	0.59	0.53

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

The results are similar apart from the degree to which the indexes load on value over the period. Value exposure under the cumulative normal approach is higher. Indeed this remains the case if value loading is calculated over rolling two-year windows as can be seen in Chart 9. The cumulative normal approach consistently achieves a higher level of loading on the given factor.

Chart 9. Rolling Two-Year Value Loading
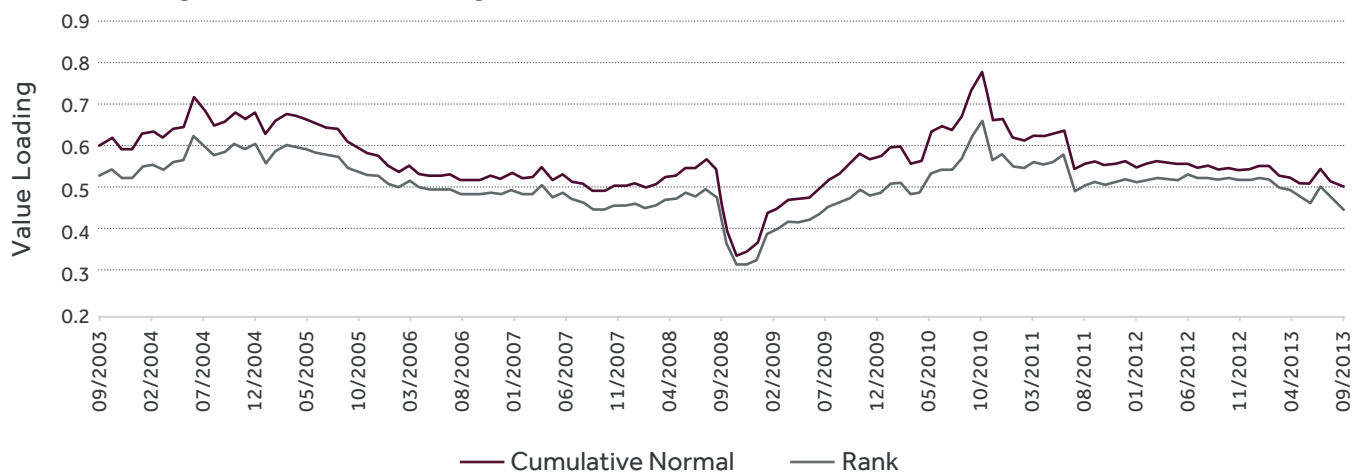
Source FTSE: September 2003 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

5.9 Narrowing of Broad Indexes

Table 10 shows the results of removing stocks with the smallest weight from a broad Earnings Yield tilted index, where the underlying index is market capitalisation weighted. The initial (broad) index has an effective number of stocks = 268 and we sequentially remove stocks to achieve a target effective number of stocks of 250, 200, 150 and 100.

Table 10. Narrowing of a Broad Index – Diversification Target: Factor Performance

	Broad factor index	Effective N = 250	Effective N = 200	Effective N = 150	Effective N = 100
Geometric Mean (%)	3.26	3.20	2.89	2.17	1.31
Volatility (%)	17.47	17.64	17.84	18.10	18.42
Volatility Reduction (%)	-1.90	-2.90	-4.06	-5.60	-7.49
Sharpe Ratio	0.19	0.18	0.16	0.12	0.07
DD (%)	-59.40	-59.52	-59.58	-59.74	-59.08
Two Way Turnover (%)	106.20	122.73	128.46	132.50	138.36
Excess (%)	1.58	1.52	1.21	0.50	-0.34
Tracking Error (%)	2.09	2.20	2.46	2.83	3.36
Information Ratio	0.76	0.69	0.49	0.18	-0.10
Alpha (%)	1.59	1.53	1.23	0.53	-0.30
Alpha T-Stat	2.84	2.63	1.90	0.73	-0.34
Beta	1.01	1.02	1.03	1.04	1.06
Average No. of Stocks	1940	1208	627	334	171
Capacity (WCR)	1.30	1.38	1.55	1.83	2.31
Value Loading	0.59	0.62	0.66	0.70	0.71

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

As smaller weighted stocks are removed, the factor exposure of the index increases. We also observe that the capacity of the indexes declines. Chart 10 illustrates the rolling two year value loading of each index through time and confirms the robustness of our observation that the application of an increasingly stringent diversification constraint results in a stronger tilt towards the factor objective. Capacity (and liquidity) may also be controlled by the application of additional constraints.

Chart 10. Rolling Two-Year Value Loading for Narrow Indexes
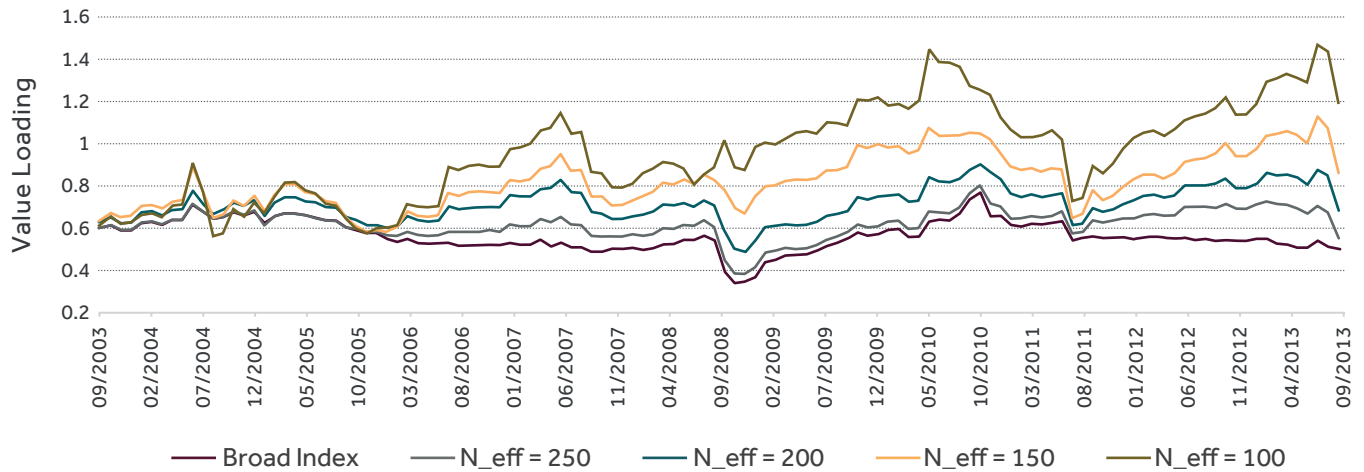
Source FTSE: September 2003 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

Table 11 shows the result of applying the same process, using both a diversification constraint with Effective N >= 200 and a capacity constraint with WCR <= 1.5. Note that both the diversification and capacity are controlled in the resulting narrow factor index.

Table 11. Narrowing Index – Diversification and Capacity Targets: Factor Performance

	FTSE developed	Broad factor index	Narrow factor index
Geometric Mean (%)	1.66	3.26	2.98
Volatility (%)	17.14	17.47	17.78
Volatility Reduction (%)		-1.90	-3.73
Sharpe Ratio	0.10	0.19	0.17
DD (%)	-58.95	-59.40	-59.60
Two Way Turnover (%)		106.20	128.06
Excess (%)		1.58	1.30
Tracking Error (%)		2.09	2.39
Information Ratio		0.76	0.54
Alpha (%)		1.59	1.32
Alpha T-Stat		2.84	2.09
Beta		1.01	1.03
Average No. of Stocks	1940	1940	763
Effective N	340	268	219
Capacity (WCR)	1.00	1.30	1.48
Value Loading	0.00	0.59	0.64

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to October 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see page 36 for important legal disclosures.

6. Conclusions

We have set out a transparent approach to the creation of long-only factor indexes that exhibit intentional exposure to factor(s) of interest by tilting an underlying index towards (or away) from a given factor. The underlying index may take any form. For example the underlying index may be market capitalisation weighted, equally-weighted, or an exotic set of alternatively weighted index weights.

At each rebalance the cross-section of factor values are converted to a set of Z-Scores with suitably truncated extreme values. The Z-Scores are then mapped to a set of Scores using the Cumulative Normal function. We have shown that the cumulative normal is superior in many respects to other commonly used functions. The Scores are used to create a tilted set of index weights by multiplying the underlying index weights by the Scores and normalizing. The resulting tilted index is guaranteed to have a greater exposure to the factor than the underlying index.

We illustrated the application of this methodology to the construction of a simple Earnings Yield Index, using both a market capitalisation and equally weighted underlying index for the FTSE Developed universe. We gradually extended the empirical approach to highlight the flexibility of the methodology; creating a composite value index, comprising Earnings Yield and Book to Price and a value-momentum tilt-tilt index of Earnings Yield and Twelve Month Price Momentum.

We outlined possible approaches to controlling the extent of any factor tilt; either through the second moment of cumulative normal mapping scheme or through the application of multiple tilts to the same factor.

We can further increase the exposure of this broad index to the factor of interest by removing stocks that contribute trivially to it while respecting capacity and diversification constraints. The addition of country or industry weight limits may also be readily incorporated in to the approach. The result is a narrow, practical index that endeavours to represent the performance of specific factor risk premia in a realisable manner.

The approach outlined is general and may be extended to incorporate tilts on multiple factors simultaneously. A factor index that is tilted towards more than one factor can be produced simply within this framework. We highlighted the importance of distinguishing between positive and negatively correlated factors. If a set of factors are positively correlated with one another a composite factor may be formed by averaging Z-Scores. Where factors are negatively correlated, such averaging may result in limited exposure to either factor (since Z-Scores offset). In this situation, tilting the underlying index towards each factor separately ensures that the resulting "tilt-tilt" index has substantial exposure to both factors.

Appendix A. Regression Analysis

Cross-sectional Regression

Consider the cross-sectional regression:

$$R_i = \alpha + \sum_{j=1}^{J} \delta_{i \subset j} \beta_j + \sum_{k=1}^{K} e_{ki} f_k + \varepsilon$$

where R_i is the return of the ith of N stocks, e_{ki} is the exposure of the ith stock to k^{th} of K factors, $\delta_{i \subset j}$ is industry (country) exposure of the ith stock to j^{th} of j industries (countries), with

$$\delta_{i \subset j} = \begin{cases} 1 \ if \ i \subset j \\ 0 \ if \ i \not\subset j \end{cases}$$

and α is an intercept, β_j is the factor return of jth industry (country), f_k is the factor return of kth factor and ε is the residual. For each stock the industry exposures meet the condition: $\sum_{j=1}^{J} \delta_{i \subset j} = 1$ which acts as a constraint on our regression. This is incorporated in the following way.

Let $\hat{R}_i$ be the return predicted by the model and W_i be some weighting scheme for stocks with $\sum_{i=1}^{N} W_i = 1$. Then employing a weighted regression through minimization of $\sum_{i=1}^{N} W_i (R_i - \hat{R}_i)^2$ implies that

$$\sum_{i=1}^{N} w_i R_i = \sum_{i=1}^{N} w_i \hat{R}_i$$

Further we can exploit freedom to set $\alpha = \sum_{i=1}^{N} W_i R_i$. Taking the weighted sum of both sides of our regression and simplifying now yields:

$$\sum_{i=1}^{N} w_i \sum_{j=1}^{J} \delta_{i \subset j} \beta_j + \sum_{i=1}^{N} w_i \sum_{k=1}^{K} e_{ki} f_k = 0$$

Reversing sums gives

$$\sum_{J=1}^{J} \hat{w}_j \beta_j + \sum_{k=1}^{K} f_k \sum_{i=1}^{N} w_i e_{ki} = 0$$

where $\hat{w}_j$ is the weight in the jth sector. This last equation is solved by setting:

$$\sum_{i=1}^{N} w_i e_{ki} = 0 \ \text{and} \ \sum_{j=1}^{J} \hat{w}_j \beta_j = 0$$

The first of these equations can be satisfied redefining the factor exposure by normalisation thus:

$$\hat{e}_{ki} = (e_{ki} - \mu_k)/\sigma_k \ where \ \mu_k = \sum_{i=1}^{N} w_i e_{ki} \ and \ \sigma_k^2 = \sum_{i=1}^{N} w_i (e_{ki} - \mu_k)^2$$

Each normalised factor exposure has mean zero and standard deviation of one.

The second equation can be satisfied by writing the return to industry (country) one as:

$$\beta_1 = -\frac{1}{\hat{w}_1} \sum_{j=2}^{J} \hat{w}_j \beta_j$$

or equivalently by defining a new exposure to industry operator:

$$\hat{\delta}_{i \subset j} = \begin{cases} -\hat{w}_j / \hat{w}_1 & \text{if } i \subset 1 \text{ and } j \neq 1 \\ 0 & \text{if } i \subset 1 \text{ and } j = 1 \\ \delta_{i \subset j} & \text{if } i \not\subset 1 \end{cases}$$

Factor and sector returns are calculated from actual returns R_i, new exposure function to industries (countries), $\hat{\delta}_{i \subset j}$ and normalised exposures to factors $\hat{e}_{ki}$ in the usual way.

Ideal Factor Indexes and Factor Loadings

We define the returns tod an "ideal factor index" X as the set of monthly factor returns f_t^x obtained through the cross-sectional regression when we ignore industries (countries), retain the market intercept and choose our value factor as a single factor.

For a given index with monthly excess returns R_t, the factor loading F_i of that index on the ith factor, is given by time series regression:

$$R_t = \alpha + \sum F_i * f_t^i + \varepsilon$$

where α is the intercept and ε is the residual.

Appendix B. Capacity, Diversification and Factor Exposure

Capacity

Let $\hat{W}_i$ be the weights of the index for which we are computing capacity and W_i the weights of the underlying market capitalisation weighted index. Then the weighted capacity ratio (WCR) is given by:

$$WCR = \sum_{i=1}^{N} \hat{W}_i * \frac{\hat{W}_i}{W_i}$$

This ratio is bounded below by one and the larger the value, the poorer the capacity.

Let M_i be the market capitalisation of ith stock. Then the weighted average market capitalisation ratio (WAMCR) is given by:

$$WAMCR = \frac{\sum_{i=1}^{N} \hat{W}_i * M_i}{\sum_{i=1}^{M} W_i * M_i}$$

This ratio is bounded below by zero and the larger its value, the better the capacity.

Both definitions maybe used to assess the capacity of an index although we prefer WCR as it does not suffer from the deficiency of implying that an index consisting of one stock with market capitalisation M has the same capacity as an index of N stocks each with market capitalisation M.

Diversification

The diversification measure we use in this document is:

$$\text{Effective N} = 1 / \sum_{i=1}^{m} W_i^2$$

where" Effective N is the effective number of stocks in they index, M the actual number of stocks and W_i the weight of ith stock in the index.

Factor Exposure

The Factor Exposure of an index is defined to be:

$$\text{Factor Exposure} = \sum_{i=1}^{N} \hat{W}_i * Z_i$$

where $\hat{W}_i$ are the weights of the index and Z_i is the cross-sectional factor Z-Score.

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