FTSE Implied Volatility Index Series

v2.4

The FTSE Implied Volatility Index Series is not, and is not intended to be, used by supervised entities in the European Union or the United Kingdom and accordingly, the European Benchmark Regulation* and the UK Benchmark Regulation# do not apply to the index series. Consequently, supervised entities within the European Union and the United Kingdom are not permitted to use any of the indices within the index series as a benchmark as set out in article 3(1)(7) of the European Benchmark Regulation.

For the avoidance of doubt, neither FTSE International Limited nor any other member of the London Stock Exchange Group plc group of companies, is the benchmark administrator (as defined in article 3(1)(6) of the European Benchmark Regulation) of the index series.

* *Regulation (EU) 2016/1011 of the European Parliament and of the Council of 8 June 2016 on indices used as benchmarks in financial instruments and financial contracts or to measure the performance of investment funds*

\# *The Benchmarks (Amendment and Transitional Provision) (EU Exit) Regulations 2019 (which amends the European benchmark regulation in the United Kingdom)*

Contents

Section 1
Introduction

1. Introduction

1.1 This document sets out the Ground Rules for the construction and management of the FTSE Implied Volatility Index (FTSE IVI) Series. Copies of the Ground Rules are available from FTSE Russell on www.lseg.com/en/ftse-russell/.

1.2 The FTSE IVI is a volatility index, which measures the interpolated 30, 60, 90, 180 and 360 day implied volatility of an underlying equity index using the index option prices.

1.3 The index is comprised of out-of-the-money put and call options from two expirations which span the period of interest. The price of each option reflects the market's expectation of future volatility.

1.4 The FTSE Implied Volatility Index Series does not take account of ESG factors in its index design.

1.5 The FTSE Implied Volatility Index Series are calculated on an end-of-day basis.

1.6 FTSE Russell

FTSE Russell is a trading name of FTSE International Limited, Frank Russell Company, FTSE Global Debt Capital Markets Limited (and its subsidiaries FTSE Global Debt Capital Markets Inc. and FTSE Fixed Income Europe Limited), FTSE Fixed Income LLC, FTSE (Beijing) Consulting Limited, Refinitiv Benchmark Services (UK) Limited, Refinitiv Limited and Beyond Ratings.

1.7 FTSE Russell hereby notifies users of the index series that it is possible that circumstances, including external events beyond the control of FTSE Russell, may necessitate changes to, or the cessation, of the index series and therefore, any financial contracts or other financial instruments that reference the index series or investment funds which use the index series to measure their performance should be able to withstand, or otherwise address the possibility of changes to, or cessation of, the index series.

1.8 Index users who choose to follow this index series or to buy products that claim to follow this index series should assess the merits of the index's rules-based methodology and take independent investment advice before investing their own or client funds. No liability whether as a result of negligence or otherwise is accepted by FTSE Russell for any losses, damages, claims and expenses suffered by any person as a result of:

– any reliance on these Ground Rules, and/or

– any inaccuracies in these Ground Rules, and/or

– any non-application or misapplication of the policies or procedures described in these Ground Rules, and/or

– any inaccuracies in the compilation of the Index or any constituent data.

Section 2
Management responsibilities

2. Management responsibilities

2.1 FTSE International Limited (FTSE)

2.1.1 FTSE is responsible for the operation and maintenance of the FTSE Implied Volatility Index Series.

2.2 Status of these Ground Rules

2.2.1 These Ground Rules set out the methodology and provide information about the publication of the FTSE Implied Volatility Index Series.

2.3 Amendments to these Ground Rules

2.3.1 These Ground Rules shall be subject to regular review (at least once a year) by FTSE Russell to ensure that they continue to best reflect the aims of the index series. The review process will include consultation on any proposed changes with the relevant FTSE Russell external advisory committees and the FTSE Russell Index Governance Board.

2.3.2 As provided for in the Statement of Principles for FTSE Russell Equity Indices, where FTSE Russell determines that the Ground Rules are silent or do not specifically and unambiguously apply to the subject matter of any decision, any decision shall be based as far as practical on the Statement of Principles. After making any such determination, FTSE Russell shall advise the market of its decision at the earliest opportunity. Any such treatment will not be considered as an exception or change to the Ground Rules, or to set a precedent for future action, but FTSE Russell will consider whether the Ground Rules should subsequently be updated to provide greater clarity.

Section 3
FTSE Russell Index policies

3. FTSE Russell Index policies

These Ground Rules should be read in conjunction with the following policy documents which can be accessed using the links below:

3.1 Statement of Principles for FTSE Russell Equity Indices (the Statement of Principles)

3.1.1 Indices need to keep abreast of changing markets and the Ground Rules cannot anticipate every eventuality. Where the Rules do not fully cover a specific event or development, FTSE Russell will determine the appropriate treatment by reference to the Statement of Principles which summarise the ethos underlying FTSE Russell's approach to index construction. The Statement of Principles is reviewed annually and any changes proposed by FTSE Russell are presented to the FTSE Russell Policy Advisory Board for discussion before approval by the FTSE Russell Index Governance Board.

The Statement of Principles can be accessed using the following link:

Statement_of_Principles.pdf

3.2 Queries and Complaints

Benchmark_Determination_Complaints_Handling_Policy.pdf

3.3 Policy for Benchmark Methodology Changes

3.3.1 Details of FTSE Russell's policy for making benchmark methodology changes can be accessed using the following link:

Policy_for_Benchmark_Methodology_Changes.pdf

3.4 FTSE Russell Governance Framework

3.4.1 To oversee its indices, FTSE Russell employs a governance framework that encompasses product, service and technology governance. The framework incorporates the London Stock Exchange Group's three lines of defence risk management framework and is designed to meet the requirements of the IOSCO Principles for Financial Benchmarks[1], the European benchmark regulation[2] and the UK benchmark regulation[3]. The FTSE Russell Governance Framework can be accessed using the following link:

FTSE_Russell_Governance_Framework.pdf

3.5 Real Time Status Definitions

3.5.1 Please refer to the following guide for details of real time status definitions for indices which are calculated in real time.

Real_Time_Status_Definitions.pdf

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[1] IOSCO Principles for Financial Benchmarks Final Report, FR07/13 July 2013.
[2] Regulation (EU) 2016/1011 of the European Parliament and of the Council of 8 June 2016 on indices used as benchmarks in financial instruments and financial contracts or to measure the performance of investment funds.
[3] The Benchmarks (Amendment and Transitional Provision) (EU Exit) Regulations 2019.

Section 4
Eligible securities

4. Eligible securities

4.1   The FTSE Implied Volatility Index Series is a set of volatility Indices that are derived from the out-of-the money put and call index options from the following Indices.

Underlying Index	FTSE Implied Volatility Index	Type	Currency	Exchange
FTSE 100 Index	FTSE 100 Implied Volatility Index	Volatility	GBP	ICE Europe
FTSE MIB Index	FTSE MIB Implied Volatility Index	Volatility	EUR	Borsa Italiana

4.2   The FTSE Implied Volatility Index uses the daily settlement prices published by the relevant exchange.

Section 5
Calculation methodology

5. Calculation methodology

5.1 Overview

5.1.1 The FTSE IVI is a volatility index, which measures the interpolated N-day implied volatility of an underlying stock index, such as the FTSE 100 or FTSE MIB. The implied volatility index is comprised of the out-of-the-money (OTM) put and call options and the price of each option reflects the market's expectation of future volatility. Like conventional indices, FTSE IVI employs rules for selecting component options and formulae to calculate index values.

5.1.2 The general formula used in the FTSE IVI calculation is:

$$\sigma^2 = \frac{2}{T}\left(1 + \log\frac{F}{K_*} - \frac{F}{K_*} + e^{rT}\int_0^{K_*}\frac{P(K)}{K^2}dK + e^{rT}\int_{K_*}^{\infty}\frac{C(K)}{K^2}dK\right) \tag{1}$$

5.1.3 Where $\sigma \times 100$ is the FTSE IVI, $P(K)$ and $C(K)$ are the put and call prices at strike K, F is the forward index level, K_* is the strike immediately below F and r is the risk free interest rate to expiration T. The above equation can be simplified as:

$$\sigma^2 = \frac{2}{T}\left(1 + \log\frac{F}{K_*} - \frac{F}{K_*} + e^{rT}\int_0^{\infty}\frac{Q(K)}{K^2}dK\right) \tag{2}$$

5.1.4 Where $Q(K)$ represents the midpoint of the bid-ask spread of an option, which is a call if $K>K_*$, a put if $K<K_*$, and the average of the put and a call if $K=K_*$.

5.1.5 Under the FTSE IVI methodology the integral in Equation (2) used to estimate σ^2 is calculated using a generalisation of Simpson's Rule, where the interval between strikes need not be equal.

5.2 Unequal interval Simpson's Rule

5.2.1 Simpson's Rule is an established method of numerical integration. However, when the interval between estimation points (i.e. strikes) is not equal, a more general version of Simpson's Rule is required.

5.2.2 Suppose there is a function $f(K)$ whose values are known at K_0, $K_0+\delta K_1$, $K_0+\delta K_1+\delta K_2$ and there is no requirement that $\delta K_1=\delta K_2$. The integral of $f(K)$ in the interval $[K_0,K_0+\delta K_1+\delta K_2]$ can be found with the Unequal-Interval Simpson's Rule:

$$S_I(K_0,\delta K_1,\delta K_2) = \int_{K_0}^{K_0+\delta K_1+\delta K_2} f(K)dK \approx \frac{\delta K_1+\delta K_2}{6\delta K_1\delta K_2}\big((2\delta K_1-\delta K_2)\delta K_2 \cdot f(K_0)$$
$$+(\delta K_1+\delta K_2)^2 \cdot f(K_0+\delta K_1)$$
$$+(2\delta K_2-\delta K_1)\delta K_1 \cdot f(K_0+\delta K_1+\delta K_2)\big) \tag{3}$$

5.2.3 In order to apply this to the integral in Equation (2), the option data is partitioned into contiguous groups of three strikes.

5.3 **End-point linear fit**

5.3.1 If there is an even number of strikes in the integral of (2) then the data for that integral cannot be partitioned into contiguous sets consisting of three points. In this situation the data is partitioned into contiguous sets of three points and a final overlapping set of two points. The set of two points are integrated using linear interpolation in the normal way for trapezoidal integration:

$$L_I(K_1,K_2) = \int_{K_1}^{K_2} f(K)dK \approx (K_2\text{-}K_1)\frac{f(K_2)+f(K_1)}{2}$$

(4)

5.4 **General integration scheme**

5.4.1 The general integration scheme employed consists of the Unequal-Interval Simpson's Rule or a mixture of the Unequal-Interval Simpson's Rule and the End-Point Linear Fit. Under this scheme integration of a set of data with n strikes requires:

$\text{Int}\frac{n\text{-}1}{2}$ applications of Unequal-Interval Simpson's Rule (Equation (3)) and

$1\text{-Mod}(n,2)$ linear interpolations at the end-points (Equation (4)).

5.4.2 Whilst the linear interpolation can be calculated at either end of the data series, the FTSE IVI methodology calculates any linear interpolation component using the lowest two strikes available.

Section 6
Step-by-step calculation guide

6. Step-by-step calculation guide

6.1 FTSE IVI is comprised of near-term and next-term put and call options. Typically these correspond to the first and second contract months of the underlying future when estimating the 30-day implied volatility, but may be any consecutive months depending on the N-day volatility to be calculated.

6.2 In order to minimize any pricing anomalies that can occur close to expiration a cut-off of one week (7 days) to expiration is used. That is, when there is less than one week to expiration of the near-term options FTSE IVI rolls to the second and third contract months.

6.3 For example, suppose FTSE IVI is being calculated for the FTSE MIB index. These index options expire on the third Friday of the month. Consequently, the second Friday in September FTSE IVI would be calculated using options expiring in September and October. However, on the following Monday, the near-term options would move from September to October and the next-term options from October to November.

6.4 The following calculation example uses FTSE MIB option prices. Options on the FTSE MIB expire at 09:05 on the third Friday of the month and there are 13 days to expiration on the near-term and 41 on the next-term. The prices used reflect those at the close of trading at 17:40.

6.5 Step 1 – Time to Expiration and Interest Rates Used in the FTSE IVI Calculation

6.5.1 The FTSE IVI calculation requires the time to expiry for each term and interest rate information.

6.5.2 The time to expiration is calculated using the number of seconds between the calculation time and the expiration time. For the purposes of the FTSE IVI calculation the precision of the time calculation is in seconds and there are 365 days in a year.

6.5.3 For simplicity, it is easiest to separate the time to expiration calculation into the time remaining on the calculation day until midnight, the time from midnight to expiration on the settlement day and the time remaining in the days between:

$$T = \frac{\left(\text{Secs}_{\text{Calculation day}} + \text{Secs}_{\text{Settlement day}} + \text{Secs}_{\text{Remaining days}}\right)}{\text{Seconds in 365 days}} \tag{5}$$

6.5.4 In the current example there are 6 hours 20 minutes from the calculation time to midnight, 9 hours 5 minutes from midnight to expiration and 13 days between calculation and settlement in the near-term and 41 days in the next-term. Hence, the time to expiration for the near-term and next-term is:

$$T_{near} = \frac{22800 + 32700 + 1123200}{31536000} = 0.037376$$

$$T_{next} = \frac{22800 + 32700 + 3542400}{31536000} = 0.114089$$

6.5.5   The interest rates used are the OIS term rates, quoted on the calculation date, which mature closest to the expiration date of the relevant options. The calculation uses 1 week, 2 week, 1 month, 2 month, 3 month, 6 month, 9 month and 12 month OIS terms. If two term rates are equally close to the expiration date then the nearest OIS term rate is used.

6.5.6   In the example, the near-term options expire in 13 days and the OIS term maturing closest to this is the 2 week rate. Similarly, for the next-term, the options expire in 41 days and the OIS term maturing closest to this is the 1 month rate. No OIS settlement day conventions or rate curve interpolation are considered.

6.5.7   The interest rates for this example are:

$$r_{near}=0.375\%$$
$$r_{next}=0.374\%$$

6.6   Step 2 – Select the options to be used in the FTSE IVI Calculation

6.6.1   Options used in the FTSE IVI calculation are those which are the out-of-the-money and have non-zero settled prices.

6.6.2   Note that the number of options used in the calculation is not fixed and will change as the underlying level and hence volatility changes.

6.7   For each contract month (i.e. both the near-term and next-term):

A. Find the forward index level:

i.   This is achieved by finding the strike at which the smallest absolute difference between call and put prices occurs. For example, in Table 1 it can be seen that this occurs for both the near-term and next-term at the 16500 strike.

Table 1: Determining the forward index level

Near-term				Next-term			
Strike	Call Settled Price	Put Settled Price	Absolute Difference	Strike	Call Settled Price	Put Sttled Price	Absolute Difference
15750	789	96	693	15000	1610	173	1437
16000	592	149	443	15500	1209	271	938
16250	419	227	192	16000	856	418	438
16500	277	335	58	16500	564	625	61
16750	170	478	308	17000	337	899	562
17000	98	655	557	17500	183	1244	1061
17250	52	859	807	18000	90	1650	1560

ii.   The forward price can be found using the formula:

$$F=Strike+e^{rT}|Call\ price-Put\ price| \qquad\qquad (6)$$

iii.   The forward index prices for the near-term and next-term, F_{near} and F_{next} are:

$$F_{near}=16500+e^{0.00375\times0.037376}(227-335)=16442$$

$$F_{next}=16500+e^{0.00374\times0.114089}(564-625)=16439$$

B. Determine the at-the-money (ATM) strike price:

i.   This is the first strike K_*, immediately below (or equal to) the index forward price. Continuing with the **example** the ATM strikes for the near-term and next-term are $K_{*,near}=16500$ and $K_{*,next}=16500$ respectively.

C. Select the options that are out-of-the-money (OTM):

i. Select the call options with strike prices $>K_*$. Starting with the first strike that is greater than K_* select the options with increasingly higher available settled strike prices.Table 2 details the near-term calls as an example.

Table 2: Near Term call inclusion

Call Strike	Settled price	Included?
16750	170	Yes
17000	98	Yes
17250	52	Yes
17500	26	Yes
17750	12	Yes
18000	5	Yes
18250	2	Yes
18500	1	Yes

ii. Now select the put options with strike prices <K_*. Starting with the first strike that is less than K_* select the options with increasingly lower available settled strike prices.Table 3 illustrates this for the near-term puts.

Table 3: Next term put inclusion

Put Strike	Settled price	Included?
10500	1	Yes
11000	3	Yes
11500	7	Yes
12000	7	Yes

iii. Finally, the put and call options for the ATM strike K_*, are selected. The price used for K_* is the average of the put and call sttled prices. For example, as shown in Table 4, in the near-term the put and call settled prices are 335 and 277 respectively, hence the price for K_* option is:

$$\text{Price}_{K_0} = \frac{277+335}{2} = 306$$

Table 4: Near term and next term OTM options

Near-term Options			Next-term Options		
Strike	Option Type	Settled Price	Strike	Option Type	Settled Price
10500	Put	1	9500	Put	6
11000	Put	3	10000	Put	7
11500	Put	7	10500	Put	8
.	.	.	.	.	.
16000	Put	149	15500	Put	271
16250	Put	227	16000	Put	418
16500	Put & Call average	306	16500	Put & Call average	594.5
16750	Call	170	17000	Call	337
17000	Call	98	17500	Call	183
.	.	.	.	.	.
18000	Call	5	19000	Call	14
18250	Call	2	19500	Call	4
18500	Call	1	20000	Call	1

D. Calculate the volatility

i. First determine the number of times Equations **(3)** and **(4)** need to be applied in each term.

ii. In the near-term there are 30 options, implying that the Unequal-Interval Simpson's Rule (Equation **(3)**) is applied:

<div align="center">Integer part of (30-1)/2 = 14 times</div>

and the linear end-point fit (Equation **(4)**) is used:

<div align="center">1- Modulus (30, 2) = 1 time.</div>

iii. Therefore, near-term options must be split into one group of two and 14 groups of three contiguous strikes.

iv. Now calculate the contribution from each group of near-term options. For example, the contribution from the group of two strikes, 10500 and 11000 found using Equation **(4)** is:

$$\frac{1}{2} \times (11000\text{-}10500) \times \left(\frac{3}{11000^2} + \frac{1}{10500^2} \right)$$

$$= 8.4659 \times 10^{-6}$$

and the contribution from strikes 11000, 11500 and 12000 found using Equation **(3)** is:

$$\frac{500+500}{6 \times 500 \times 500} \times \left((2 \times 500\text{-}500) \times \frac{500 \times 3}{11000^2} \right.$$

$$+ (500+500)^2 \times \frac{7}{11500^2}$$

$$\left. + (2 \times 500\text{-}500) \times \frac{500 \times 7}{12000^2} \right)$$

$$= 4.7521 \times 10^{-5}$$

v. A similar calculation is required for all the other groups of three strikes in the near-term. Table 5 below summarises the results for the near-term options.

Table 5: Near term strike grouping

Strike group			Strike Differences		Contribution
K_0	K_1	K_2	δK_1	δK_2	
.	10500	11000	.	.	8.4659x10-6
11000	11500	12000	500	500	4.7521x10-5
12000	12250	12500	250	250	2.3333x10-5
12500	12750	13000	250	250	2.1538x10-5
13000	13250	13500	250	250	2.0400x10-5
13500	13750	14000	250	250	2.3877x10-5
14000	14250	14500	250	250	2.8254x10-5
14500	14750	15000	250	250	4.6958x10-5
15000	15250	15500	250	250	8.9208x10-5
15500	15750	16000	250	250	1.9901x10-4
16000	16250	16500	250	250	4.2871x10-4
16500	16750	17000	250	250	3.2390x10-4
17000	17250	17500	250	250	9.3584x10-5
17500	17750	18000	250	250	2.1057x10-5
18000	18250	18500	250	250	3.5311x10-6
Near-term total contribution=sum of all contributions					1.3792x10-3

vi. Next calculate the term preceding the integral in equation **(2)**:

$$1+\log\frac{F_{near}}{K_{*,near}}-\frac{F_{near}}{K_{*,near}}=1+\log\left(\frac{16558}{16500}\right)-\frac{16558}{16500}=-6.1637\times10^{-6}$$

vii. Finally, the near-term calculation can be completed:

$$\sigma^2_{near}=\frac{2}{T_{near}}\left(1+\log\frac{F_{near}}{K_{*,near}}-\frac{F_{near}}{K_{*,near}}+e^{r_{near}T_{near}}\left[\sum_i S_l(K_0,\delta K_1,\delta K_2)+L_l(K_1,K_2)\right]\right)$$

$$=\frac{2}{0.037376}\left(-6.1637\times10^{-6}+e^{0.00375\times0.037376}\times1.3792\times10^{-3}\right)$$

$$=7.3484\times10^{-2}$$

viii. In the next-term there are 22 options, which means there 10 groups of three options that are used with Equation **(3)** and one group of two options used with equation **(4)**. The results for the next-term are given in Table 6 below.

Table 6: Next term strike grouping

Strike Group			Strike Differences		Contribution
K_0	K_1	K_2	δK_1	δK_2	
.	9500	10000	.	.	3.4120x10-5
10000	10500	11000	500	500	7.2438x10-5
11000	11500	12000	500	500	8.5208x10-5
12000	12500	13000	500	500	1.3228x10-4
13000	13500	14000	500	500	2.4470x10-4
14000	14500	15000	500	500	5.3476x10-4

Strike Group			Strike Differences		Contribution
K_0	K_1	K_2	δK_1	δK_2	
15000	15500	16000	500	500	1.1523×10^{-3}
16000	16500	17000	500	500	1.9223×10^{-3}
17000	17500	18000	500	500	6.3901×10^{-4}
18000	18500	19000	500	500	1.2678×10^{-4}
19000	19500	20000	500	500	1.3893×10^{-5}
Next-term total contribution=sum of all contributions					4.9577×10^{-3}

Similarly:

$$1+\log\frac{F_{next}}{K_{*,next}} - \frac{F_{next}}{K_{*,next}} = 1 + \log\left(\frac{16561}{16500}\right) - \frac{16561}{16500} = -6.1493 \times 10^{-6}$$

and the completed next-term calculation is:

$$\sigma_{next}^2 = \frac{2}{T_{next}}\left(1+\log\frac{F_{next}}{K_{*,next}} - \frac{F_{next}}{K_{*,next}} + e^{r_{next}T_{next}}\left[\sum_i S_l(K_0,\delta K_1,\delta K_2) + L_l(K_1,K_2)\right]\right)$$

$$= \frac{2}{0.114089}\left(-6.1493 \times 10^{-6} + e^{0.00374 \times 0.114089} \times 4.9577 \times 10^{-3}\right)$$

$$= 8.6828 \times 10^{-2}$$

ix. The N-day FTSE IVI value can now be calculated. In this example the FTSE IVI value is a 30-day interpolated value and this interpolation requires the number of seconds to the settlement of the near-term and next-term options. The number of seconds in one year and in 30 days:

$$S_{near}=1178700$$
$$S_{next}=3597900$$
$$S_{year}=31536000$$
$$S_{30\ days}=2592000$$

x. With these the interpolated 30-day volatility is calculated using the following:

$$FTSE\ IVI=100\times\sqrt{\frac{1}{S_{30\ days}}\left(\frac{S_{next}-S_{30\ days}}{S_{next}-S_{near}}\times S_{near}\times\sigma_{near}^2 + \frac{S_{30\ days}-S_{near}}{S_{next}-S_{near}}\times S_{next}\times\sigma_{next}^2\right)} \qquad (7)$$

xi. Hence, the 30 day FTSE IVI is:

$$FTSE\ IVI=100\times\sqrt{\frac{1}{2592000}\times\left(\frac{3597900-2592000}{3597900-1178700}\times1178700\times7.3484\times10^{-2} + \frac{2592000-1178700}{3597900-1178700}\times3597900\times8.6828\times10^{-2}\right)}$$

=29.03

Appendix A
Further information

A Glossary of Terms used in FTSE Russell's Ground Rule documents can be found using the following link:

Glossary.pdf

Further information on the FTSE Implied Volatility Index Series is available from FTSE Russell.

For contact details please visit the FTSE Russell website or contact FTSE Russell client services at info@ftserussell.com.

Website: www.lseg.com/en/ftse-russell/

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